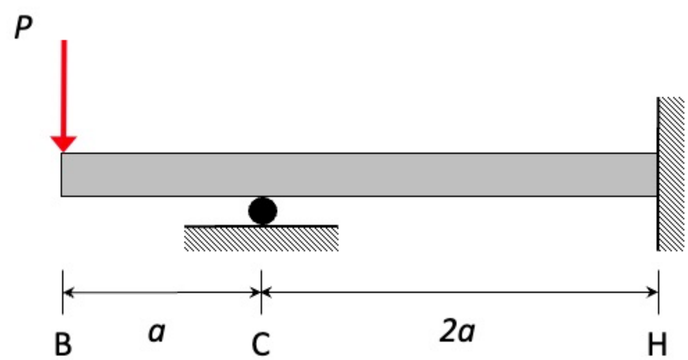
ME 323: Mechanics of Materials **_Homework Set H20_**

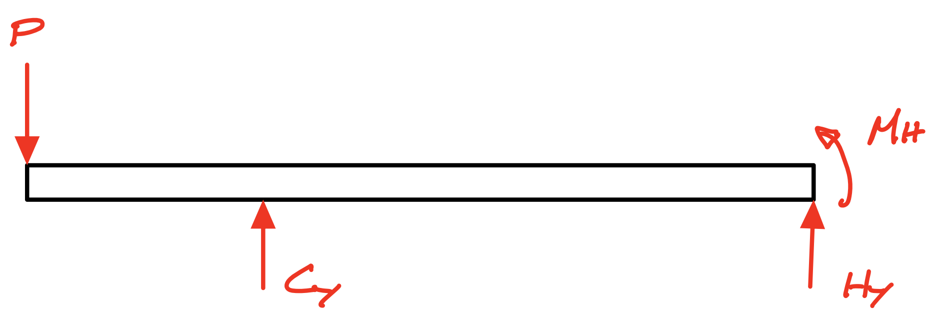
Summer 2024 **_Assigned/Due: July 9/July 11_**

Consider the beam shown below that is supported by a roller at C, and by a fixed wall at end H. A end load P acts at B. The cross section has a second area moment of I and is made up of a material having a Young's modulus of E. It is desired to determine the reactions at supports C and H using Castigliano's method. To this end:

a) Draw a free body diagram of the entire beam and write down the equilibrium equations. Show that the problem is statically indeterminate.
b) Choose an appropriate set of redundant constraint force(s) from your FBD above.
c) Write down the strain energy expression for the beam. You may neglect the contributions to the strain energy from shear.
d) Use Castigliano's method to determine the reactions at C and H.



a) Equilibrium



$$\Sigma M_H = M_H - C_y(2a) + P(3a) = 0$$

(1) $\quad \hookrightarrow M_H = -3Pa + 2C_y a$

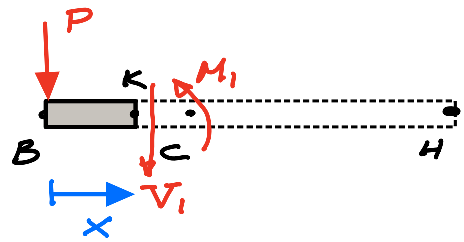
$$\Sigma F_y = -P + C_y + H_y = 0$$

(2) $\quad \hookrightarrow H_y = P - C_y$

Two equations, three unknowns $\Rightarrow$ INDETERMINATE

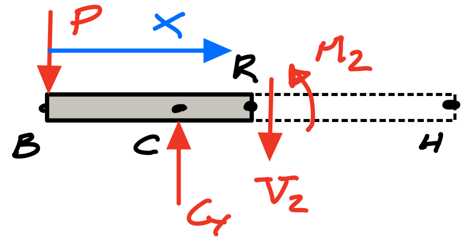
b) Choose C_y as the redundant constraint force.

c) <u>Energy expression</u>



$\underline{BC}$: $\sum M_K = M_1 + Px = 0$

$\hookrightarrow M_1(x) = -Px$



$\underline{CH}$: $\sum M_R = M_2 + Px - C_y(x-a)$

$\hookrightarrow M_2(x) = -Px + C_y(x-a)$

$$\therefore\ U = \frac{1}{2EI}\left[\int_0^a M_1^2\,dx + \int_a^{3a} M_2^2\,dx\right]$$

d) <u>Castigliano</u>

$$\frac{\partial U}{\partial C_y} = 0 = \cancel{\frac{1}{EI}}\left[\int_0^a M_1 \cancelto{0}{\frac{\partial M_1}{\partial C_y}}\,dx + \int_a^{3a} M_2 \frac{\partial M_2}{\partial C_y}\,dx\right]$$

$$= \int_a^{3a} \left[-Px + C_y(x-a)\right](x-a)\,dx$$

$$= \int_a^{3a} \left[-P(x^2-ax) + C_y(x^2-2ax+a^2)\right]dx$$

$$= \Big[P\left(-\tfrac{1}{3}x^3 + \tfrac{1}{2}ax^2\right)$$

$$+ C_y\left(\tfrac{1}{3}x^3 - ax^2 + a^2x\right)\Big]_a^{3a}$$

$$= P\left\{-\tfrac{1}{3}\left[(3a)^3 - a^3\right] + \tfrac{1}{2}a\left[(3a)^2 - a^2\right]\right\}$$

$$+ C_y\left\{\tfrac{1}{3}\left[(3a)^3 - a^3\right] - a\left[(3a)^2 - a^2\right] + a^2\left[3a - a\right]\right\}$$

$$= -\tfrac{14}{3}Pa^3 + \tfrac{8}{3}C_y a^3$$

$\hookrightarrow$

$$C_y = \tfrac{7}{4}P$$

(1) $\Rightarrow M_H = -3Pa + 2\left(\tfrac{7}{4}P\right)a = \tfrac{1}{2}Pa$

(2) $\Rightarrow H_y = P - C_y = -\tfrac{3}{4}P$