Consider the single-node truss shown below.

a) Draw a free body diagram of joint C and write down the equilibrium equations for the joint. Show that the problem is statically indeterminate.
b) Choose an appropriate set of redundant constraint force(s) from your FBD above.
c) Write down the strain energy expression for the truss.
d) Use Castigliano's method to determine the load carried by the three members of the truss.
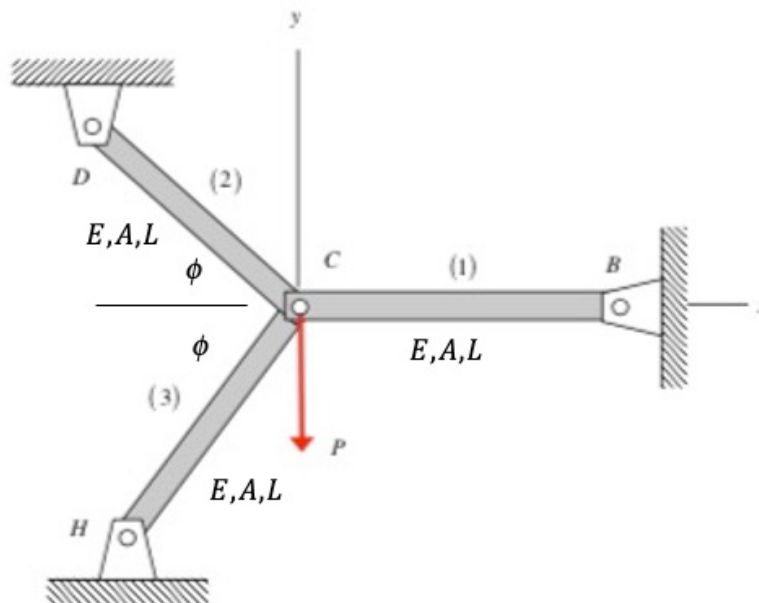
Equilibrium

$$\sum F_x = -F_2 \cos\phi - F_3 \cos\phi + F_1 = 0$$
$$\sum F_y = F_2 \sin\phi - F_3 \sin\phi - P = 0$$

2 equations / 3 unknowns $\Rightarrow$
INDETERMINATE w/ 1 redundant reaction

Choose F_3 as the redundant reaction.

$(2) \Rightarrow F_2 = F_3 + \dfrac{P}{\sin\phi}$

$(1) \Rightarrow F_1 = F_2 \cos\phi + F_3 \cos\phi$
$\qquad = 2 F_3 \cos\phi + P \cot\phi$

Strain energy

$$U = U_1 + U_2 + U_3$$

w/

$$U_1 = \frac{1}{2} \frac{F_1^2 L}{EA} = \frac{1}{2}(2F_3\cos\phi + P\cot\phi)^2 \frac{L}{EA}$$

$$U_2 = \frac{1}{2} \frac{F_2^2 L}{EA} = \frac{1}{2}\left(F_3 + \frac{P}{\sin\phi}\right)^2 \frac{L}{EA}$$

$$U_3 = \frac{1}{2} \frac{F_3^2 L}{EA}$$

Castigliano's 2nd Theorem

$$0 = \frac{\partial U}{\partial F_3} = (2F_3\cos\phi + P\cot\phi)(2\cos\phi)\frac{L}{EA} + \left(F_3 + \frac{P}{\sin\phi}\right)\frac{L}{EA} + \frac{F_3 L}{EA}$$

$$= 2(1 + 2\cos^2\phi)F_3 + \left(\frac{1 + 2\cos^2\phi}{\sin\phi}\right)P$$

$$\hookrightarrow F_3 = -\frac{1}{2}\frac{P}{\sin\phi} \quad (C)$$

 $\xleftarrow{\hspace{6cm}}$ F_3

$(2) \Rightarrow F_2 = F_3 + \frac{P}{\sin\phi} = \frac{1}{2}\frac{P}{\sin\phi}$ (T) $\xleftarrow{\hspace{3cm}}$ F_2

$(1) \Rightarrow F_1 = 2F_3\cos\phi + P\cot\phi = 0$ $\xleftarrow{\hspace{3cm}}$ F_1