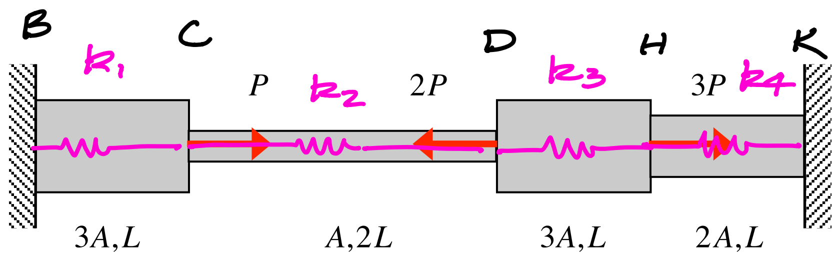
SOLUTION

ME 323: Mechanics of Materials **Homework H25**

Summer 2021

Consider the axially-loaded rod shown below. The material is constant throughout, having a Young's modulus of E. Develop the equilibrium equations for a four-element finite element model for the rod. Enforce the boundary conditions on your equilibrium equations. You do not need to solve the resulting equations.

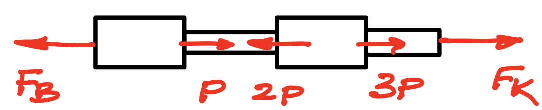


$$k_1 = \frac{E(3A)}{L} = 3\frac{EA}{L}$$

$$k_2 = \frac{EA}{2L} = \frac{1}{2}\frac{EA}{L}$$

$$k_3 = \frac{E(3A)}{L} = 3\frac{EA}{L}$$

$$k_4 = \frac{E(2A)}{L} = 2\frac{EA}{L}$$



$\therefore$

$$[K] = \begin{bmatrix} 3 & -3 & & & \\ -3 & \frac{7}{2} & -\frac{1}{2} & & \\ & -\frac{1}{2} & \frac{7}{2} & -3 & \\ & & -3 & 5 & -2 \\ & & & -2 & 2 \end{bmatrix} \frac{EA}{L}$$

$$\{F\} = \begin{Bmatrix} -F_B \\ P \\ -2P \\ 3P \\ F_K \end{Bmatrix}$$

To enforce BCs, strike out the 1^{st} & 5^{th} rows of $[K]$ & $\{F\}$, along with the 1^{st} & 5^{th} columns of $[K]$:

$$[K] = \begin{bmatrix} \frac{7}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{7}{2} & -3 \\ 0 & -3 & 5 \end{bmatrix} \frac{EA}{L}$$

$$\{F\} = \begin{Bmatrix} P \\ -2P \\ 3P \end{Bmatrix}$$

$\therefore$ the equilibrium equations are:

$$[K]\{u\} = \{F\}$$

Solve for displacements:

$$\{u\} = [K]^{-1}\{F\}$$