ME 323: Mechanics of Materials **Homework 11**

Spring 2025 **Due: April 25, 2025**

Problem 1 A fixed beam BD is subjected to a point load P at D, as shown in figure (a). The beam has an I-beam cross section as shown in figure (b), and is composed of a material of Young's modulus E.

(a) Construct the shear force and bending moment diagrams for the beam.
(b) Determine the stresses induced at point Q, as shown in figure (a) and (b), and draw a three-dimensional stress element of Q (on figure (c)).
(c) Draw the Mohr's circle for the stress state at point Q and determine the principal stresses and the absolute maximum shear stress $\tau_{max,abs}$.

P = 20 kN, L = 1.5 m, b = 0.5 m, E = 200 GPa.

Figure (a)

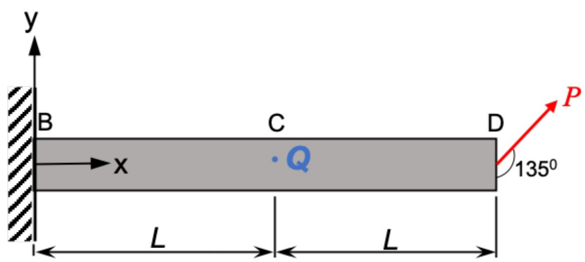


Figure (b)

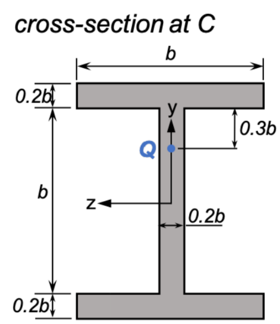
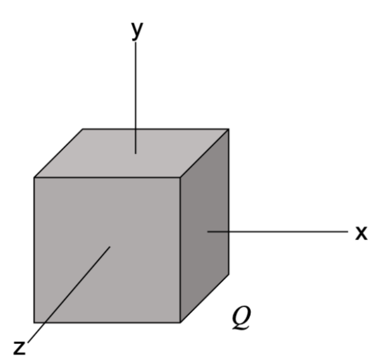


Figure (c)



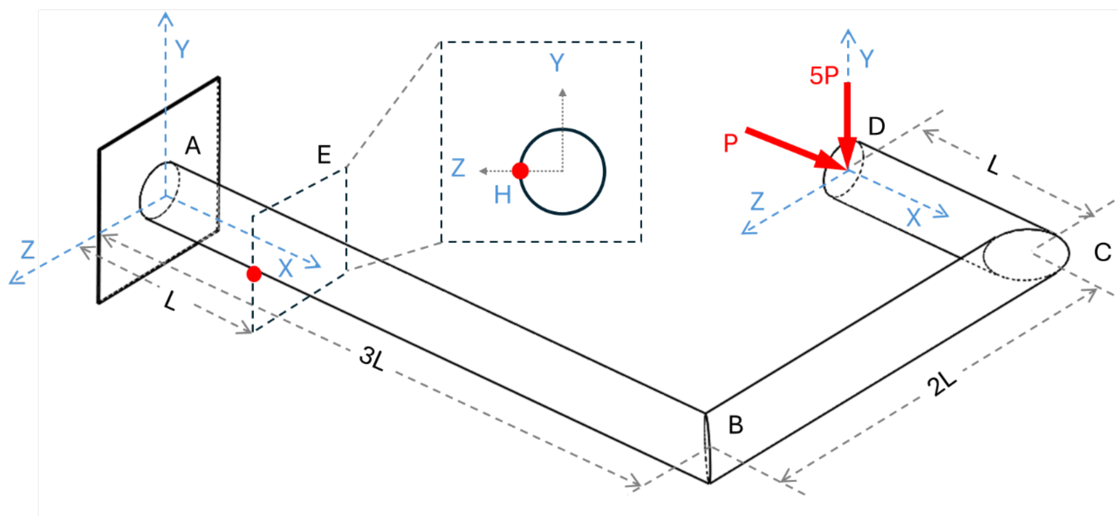
Problem 2 The cylindrical structure shown in the figure below is subjected to loads P and 5P acting along the X and Y directions at point D at the free end, respectively. The diameter of the pipe is d, and the dimensions of AB, BC and CD are 3L, 2L and L, respectively. The elastic modulus is E. Determine the following:

(a) Use the table method to show the load applied at the cutting plane E at a distance of L from A. Classify the forces as either normal or shear force, and the moments as either bending or torsion moment.

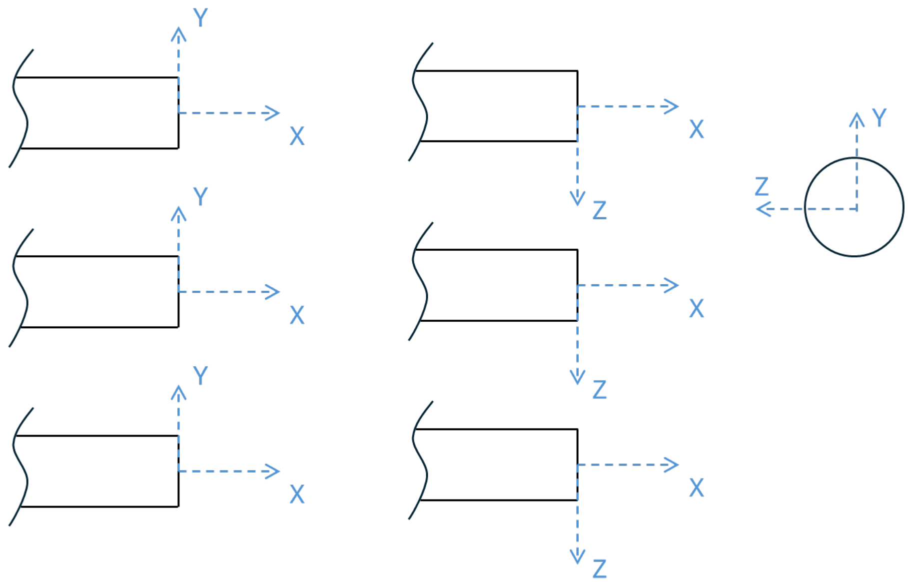
(b) Draw the stress profiles developed in the face normal to the X-axis created by the cut of the cylindrical structure with the plane E. Use the sections provided in the next page to draw the stress profiles. (normal stresses and shear stresses)

(c) Determine the stresses at point H and show the state of stress in the infinitesimal volume element given in the next page.

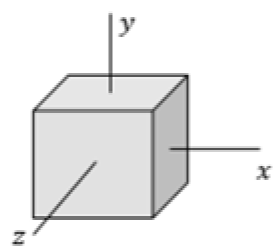
(d) Determine the principal stresses and absolute maximum stress at point H.



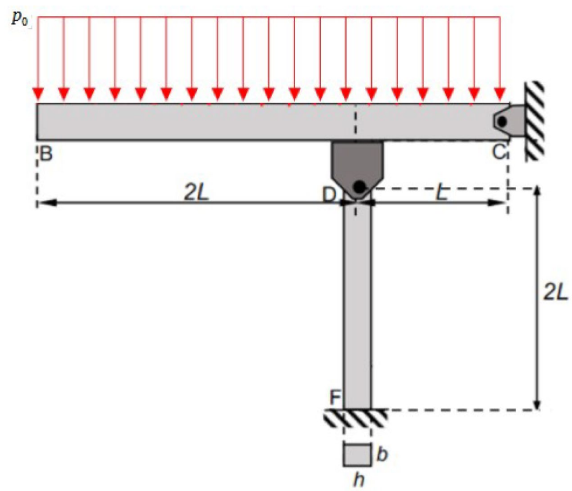
For (b):



For (c)

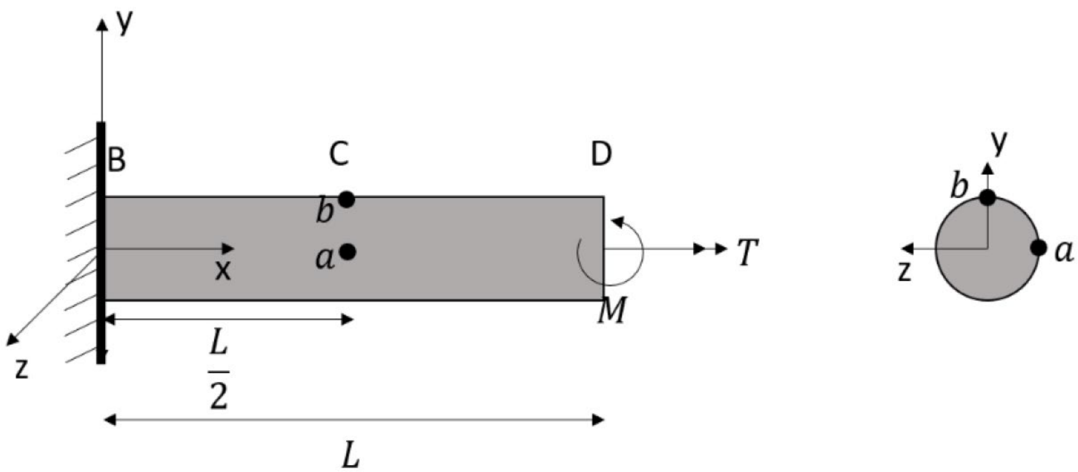


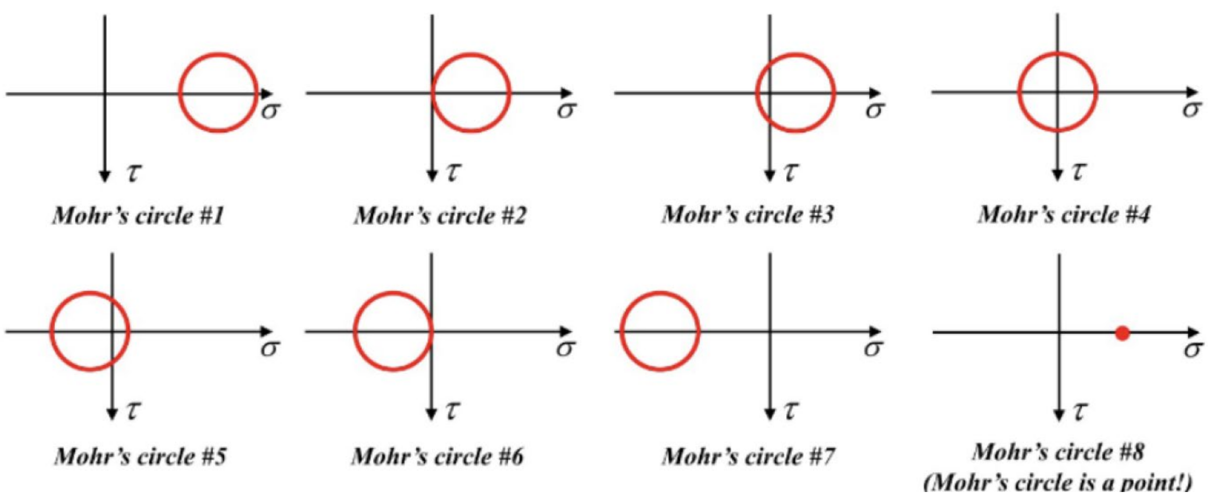
Problem 3 A rigid beam BC is pinned to a wall at C and is supported by a flexible column DF as shown in the figure below, where DF has a fixed support at end F. The beam is acted upon by a uniformly distributed load over its entire length. Column DF has a uniform rectangular cross section b×h and is composed of a material of Young's modulus E = 200 GPa. Calculate the maximum uniformly distributed load p_0 such that column DF does not buckle as predicted by Euler's buckling theory. Use L=1 m, h = 4cm, b = 2 cm. (Hint: Consider both in-plane and out-of-plane buckling for the column.)



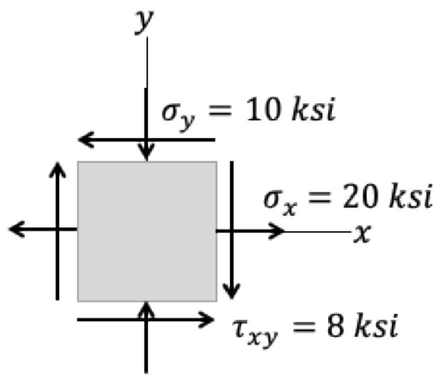
Problem 4

4.1 A moment M (about positive z) and torque T (about positive x) are applied to a circular rod as shown below. Choose the correct in-plane Mohr's circle, from the given options, for the stress states at Point a and Point b. (Note the location of Point a is at (L/2, 0, R) where R is the radius of the cross section.)





4.2 Consider the state of plane stress as shown below.



(1) What is the factor of safety using the maximum shear stress theory for a ductile material with yield strength 32 ksi? Please justify your answer.
   (a) 0.82
   (b) 1.88
   (c) 0.94
   (d) 1.1

(2) What is the factor of safety using the maximum distortion energy theory for a ductile material with yield strength 32 ksi? Please justify your answer.
   (a) 1.1
   (b) 0.94
   (c) 1.88
   (d) 0.82

(3) Whether or not failure is predicted using Mohr's failure criterion for a brittle material with ultimate compressive strength 90 ksi and ultimate tensile strength 30 ksi. Please justify your answer.
   (a) Failure is predicted.
   (b) Failure is not predicted.