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**Problem 1 (10 points):**

The propped cantilever ABC shown in the figure is subjected to a distributed load $w_0$ over half of its length. The beam of constant E,I is supported by an elastic rod BD with cross sectional area A.

a) Use the Castigliano's theorem to determine the **deflection of point B** on the beam.
b) Plot M(x) and V(x) across the beam AC. Mark the critical values in the diagrams such as the maximum and minimum values, and locations of their zero values.

**Ignore the shear energy due to bending in your analysis. Express answers in terms of given parameters E,I,L,A,$w_0$**

**Assume $I > AL^2$ for graphing V(x) and M(x).**

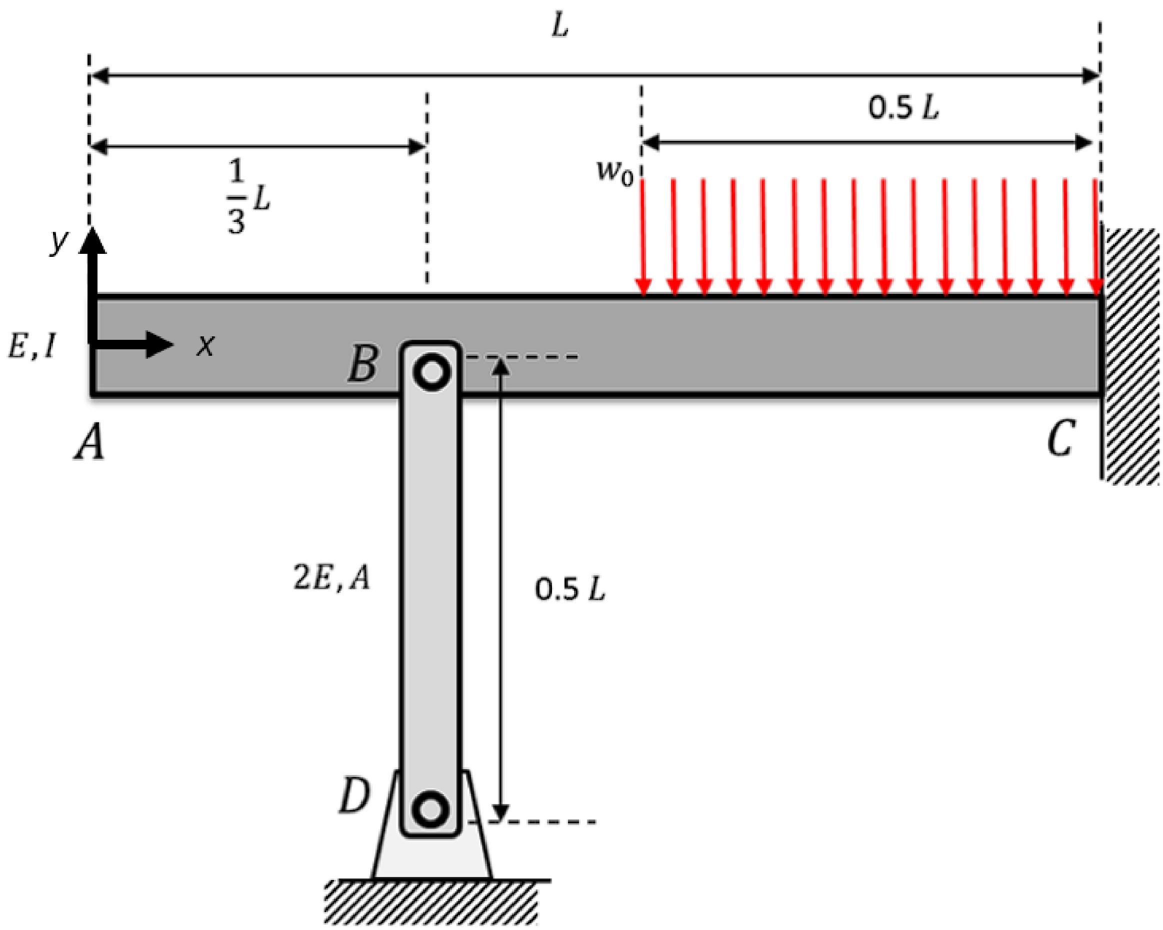



**Figure 1:** Beam for Problem 1

**Problem 2 (10 points):**

A three-segment rod AD is fixed to walls at A and D. An external load $2P$ is applied at C. The properties are shown in the figure.

(1) Use three finite elements (one element per segment), write down the stiffness matrix $K$ and the force vector $F$.

(2) Enforcing the boundary conditions, write the reduced system of equations and solve for the displacements at B and C.

(3) Find the reactions at A and D.

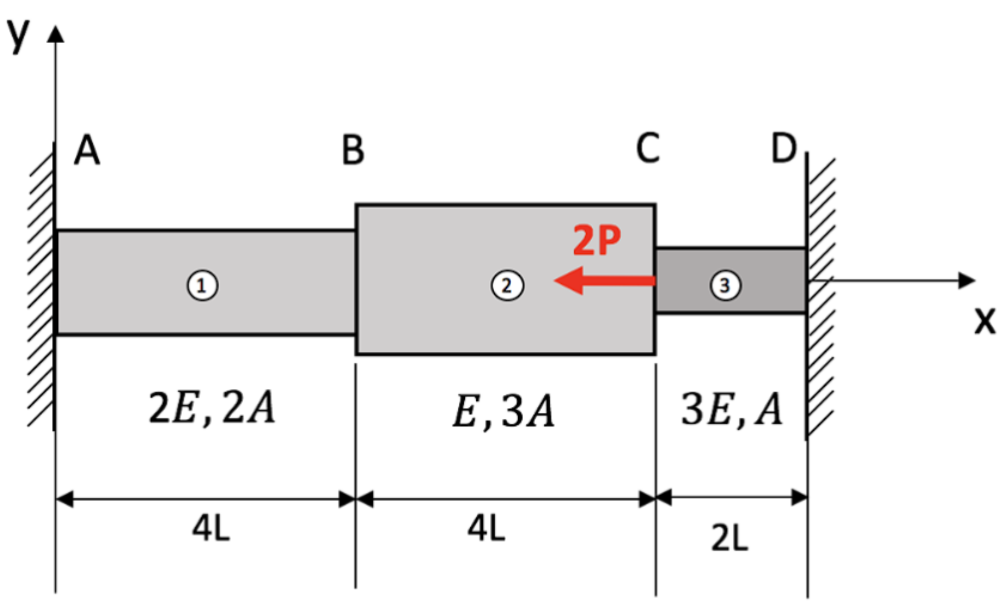



**Figure 2**: 3 rod structure for FEA in Problem 2

**Problem 3 (10 points):**

Two segments, AB and BC, with a thin-walled hollow circular cross section of outer diameter $a$ and inner diameter $0.8a$, are welded together at B to form the L-shaped frame ABC shown in the figure below. Use Castigliano's second theorem to determine

(a) Reaction force at B.

(b) Slope θ at point A.

Use the following data in your analysis: E = 280 GPa, G = 120 GPa, $a$ = 20 mm, $M_0$ =1000 Nm.

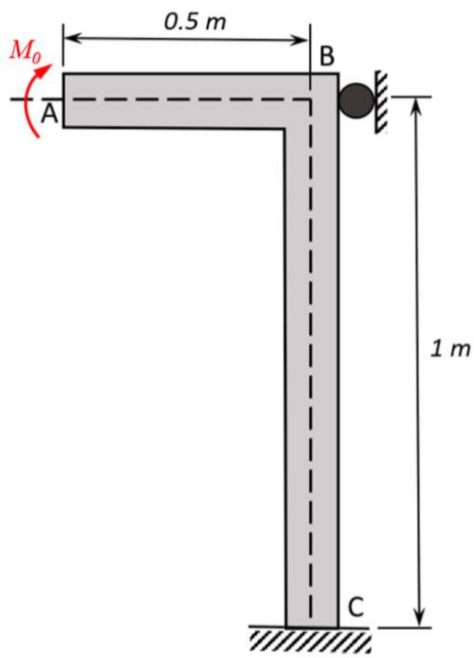



**Figure 3:** L-shaped beam for problem 3

**Problem 4 (2.5 points + 2.5 points):**

4.1. Considering the stepped rod below with the ends fixed. 2 axial forces are applied at C and D. If you were to use 3 finite elements to solve this scenario what would the stiffness matrix $[K_\phi]$ look like?

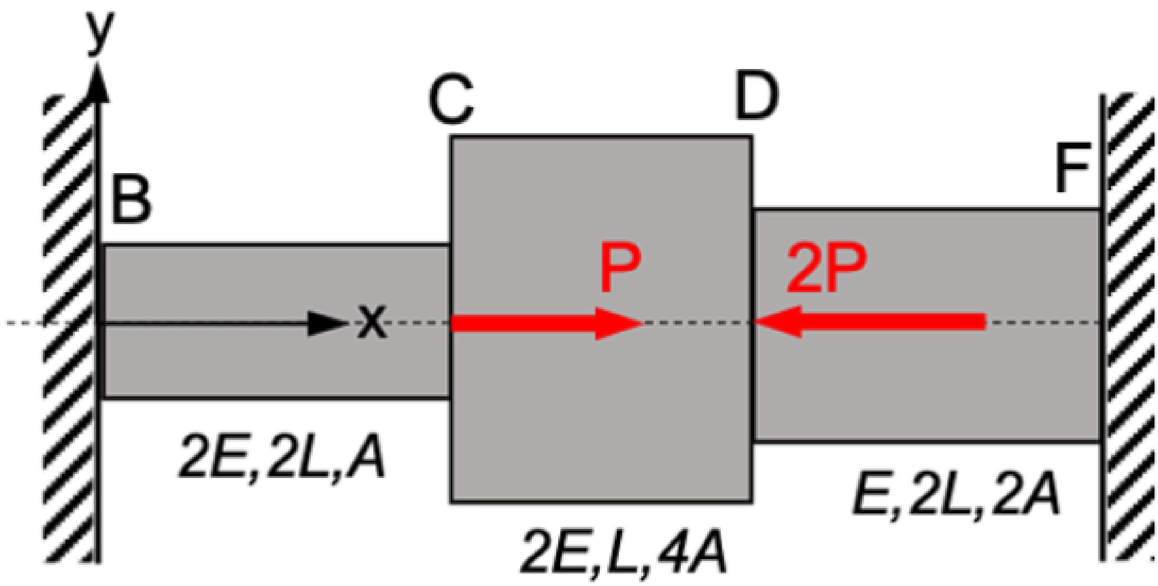



**Figure 4.1:** Stepped rod case for problem 4.1

a) $\frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 9 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

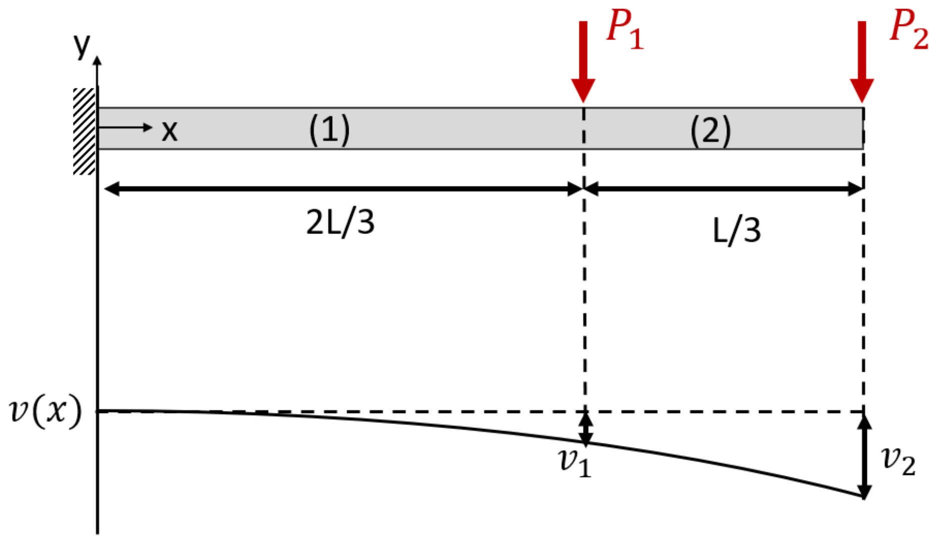
b) $\frac{EA}{L}\begin{bmatrix} 1 & -1 & -8 \\ -3 & 9 & -1 \\ -8 & -1 & 1 \end{bmatrix}$

c) $\frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 9 & -8 & 0 \\ 0 & -8 & 9 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

d) $\frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 8 & -8 & 0 \\ 0 & -8 & 8 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

e) None of the above

4.2 A cantilever beam has 2 loads $P_1$ and $P_2$ acting on it as shown below:




If we were to apply Castigliano's theorem to obtain deflection $v_1$, what would the equation look like (ignore shear energy due to bending):

a) $v_1 = \frac{\delta U}{\delta P_2} = \frac{1}{EI}\int_0^L M_1\left(\frac{\delta M_1}{\delta P_2}\right)dx + \frac{1}{EI}\int_0^L M_2\left(\frac{\delta M_2}{\delta P_2}\right)dx$

b) $v_1 = \frac{\delta U}{\delta P_1} = \frac{1}{EI}\int_0^{\frac{2L}{3}} M_1\left(\frac{\delta M_1}{\delta P_1}\right)dx + \frac{1}{EI}\int_{\frac{2L}{3}}^L M_2\left(\frac{\delta M_2}{\delta P_1}\right)dx$

c) $v_1 = \frac{\delta U}{\delta P_1} = \frac{1}{EI}\int_0^{\frac{2L}{3}} M_1\left(\frac{\delta M_1}{\delta P_1}\right)dx + \frac{1}{EI}\int_0^L M_2\left(\frac{\delta M_2}{\delta P_1}\right)dx$

d) $v_1 = \frac{\delta U}{\delta P_2} = \frac{1}{EI}\int_0^{\frac{2L}{3}} M_1\left(\frac{\delta M_1}{\delta P_2}\right)dx + \frac{1}{EI}\int_{\frac{2L}{3}}^L M_2\left(\frac{\delta M_2}{\delta P_2}\right)dx$

e) None of the above