February 23, 2022

INSTRUCTIONS

Begin each problem in the space provided. If additional space is required, use the paper provided to you.

Work appearing on the backside of any exam page will NOT be graded.

If your solution does not follow a logical thought process, it will be assumed to be in error.

PROBLEM No. 1 (25 points)

Problem 1 consists of 10 questions. Each question is worth 2.5 points.

(a) A lighting truss in a theater is suspended from the ceiling by wire ropes. The truss hangs above the audience and is designed to hold 10 lights.

The factor of safety for the wire ropes is most likely:

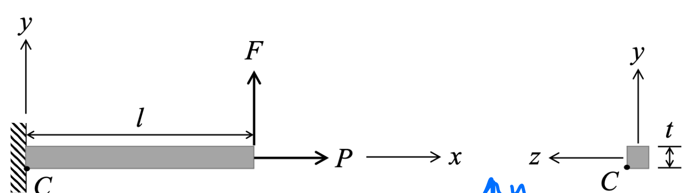
- ○ 1.2
- ○ 2
- ○ 5
- ● 10

note: other responses counted as correct, depending on justification

In a few words, justify your answer.

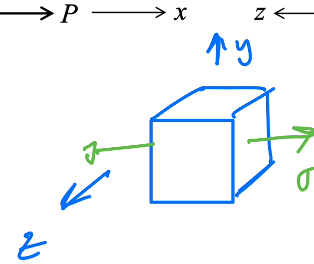
a falling truss would be catestrophic and if it is positioned over the audience, it might be difficult to inspect the wires for signs of failure.

(b) The square cantilevered beam is acted on by loads F and P.



Location C is a state of plane stress.

- ● True
- ○ False



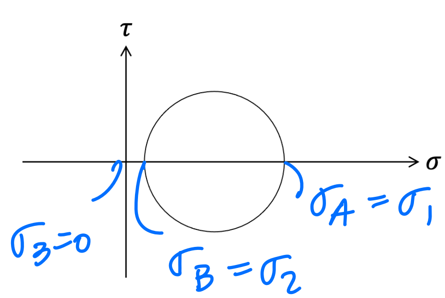
$$\sigma_x = \frac{Flt/2}{\frac{1}{12}t^4} + \frac{P}{t^2}$$

(c) The Mohr's circle for a particular state of plane stress is shown below. The part is made of a brittle material.

The factor of safety using the Brittle Coulomb Mohr (BCM) theory is 1.5. The factor of safety using the Modified Mohr (MM) theory is most likely:

- ○ 0.9
- ○ 1.3
- ● 1.5
- ○ 1.7
- ○ 3.1

From Figure 5-19 the BCM and MM failure criteria are identical when $\sigma_A > 0$ and $\sigma_B \geq 0$.

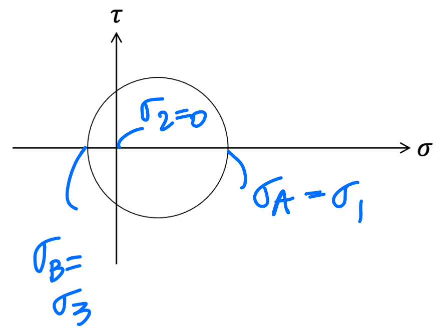


(d) The Mohr's circle for a particular state of plane stress is shown below. The part is made of a ductile material.

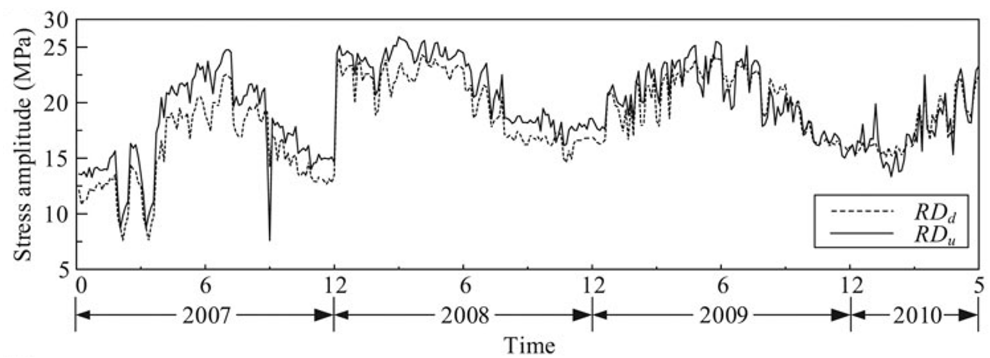
The factor of safety using the Maximum Shear Stress (MSS) theory is 1.5. The factor of safety using the Distortion Energy (DE) theory is most likely:

- ◯ 0.9
- ◯ 1.3
- ◯ 1.5
- ● 1.7
- ◯ 3.1

From Figure 5-15 the DE theory is less conservative → factor of safety will be slightly larger



(e) The deck of a bridge is subjected to the stress-time pattern shown below, representing three and a half years of traffic.



The most applicable fatigue method is:

- ◯ Stress-life
- ◯ Strain-life
- ◯ Linear elastic fracture mechanics

In a few words, justify your answer.

LEFM would be used if an inspection schedule is followed to detect cracks.

Strain-life could be used in case the bridge experiences plastic deformation.

Stress life would not be applicable because the stress is not predictable and repeatable.

(f) A 1-inch diameter rotating steel shaft is loaded in bending. The fully corrected endurance limit is $S_e = 26.5$ kpsi.

The shaft diameter is increased from 1 inch to 1.5 inches. All other conditions are kept the same. The fully corrected endurance limit is now:

- [x] $S_e = 25.4$ kpsi
- [ ] $S_e = 26.5$ kpsi
- [ ] $S_e = 27.6$ kpsi
- [ ] Cannot be determined from the given information

changing the diameter would impact k_b.

$k_b = 0.879(1\text{in})^{-0.107} = 0.879$ $\quad k_b = 0.879(1.5)^{-0.107} = 0.842$

$S_e = 26.5\text{kpsi} \cdot \frac{0.842}{0.879} = 25.4$ kpsi

(g) A colleague hands you a steel part and asks you to quickly calculate the part's factor of safety for infinite life (n_f) using the Goodman criterion.

You cannot tell if the part's surface is ground or machined.

Which surface finish will give you the more conservative prediction for n_f?

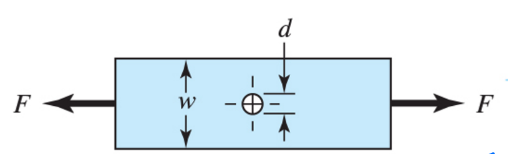
- [ ] A ground surface is more conservative.
- [x] A machined surface is more conservative.

In a few words, justify your answer.

From Figure 6-24 k_a will be smaller for machined than ground. smaller $k_a \rightarrow$ smaller $S_e \rightarrow$ lower n_f. $\rightarrow$ more conservative

(h) A bar with width $w = 10$ inches and thickness $t = 1$ inch includes a transverse hole with diameter $d = 1$ inch. The bar is made of a brittle material and failed when the static load increased to $F = 1000$ lbf.

The hole diameter is increased to $d = 2$ inches, while the bar width, thickness, and material are unchanged.



The static axial load expected to fail the bar is now:

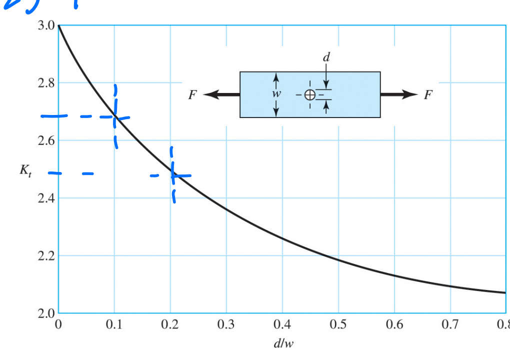
$\sigma_{max} = (K_t \sigma_0)_{original} = (K_t \sigma_0)_{updated}$ $\quad \sigma_0 = \frac{F}{(w-d)t}$

$= 2.7 \frac{1000\text{ lbf}}{(10-1)\cdot 1} = 2.5 \cdot \frac{F_2}{(10-2)\cdot 1}$ $\rightarrow F_2 = 960$ lbf

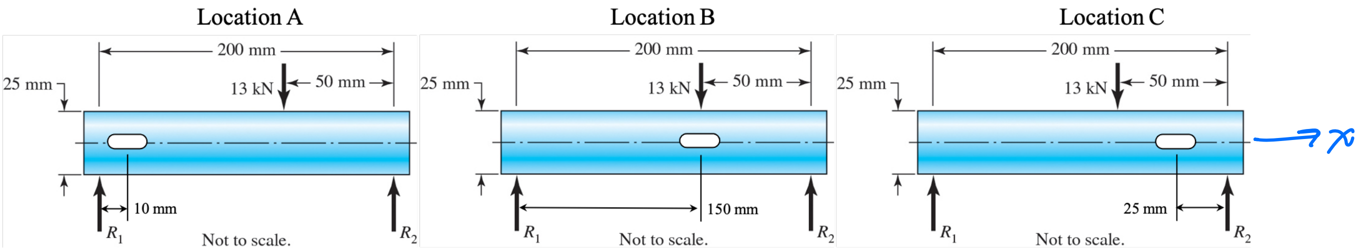
- [ ] $F = 920$ lbf
- [x] $F = 960$ lbf
- [ ] $F = 1000$ lbf
- [ ] $F = 1040$ lbf
- [ ] $F = 1080$ lbf
- [ ] Cannot be determined from the given information

K_t for $d = 1$ in $= 2.7$

K_t for $d = 2$ in $= 2.5$

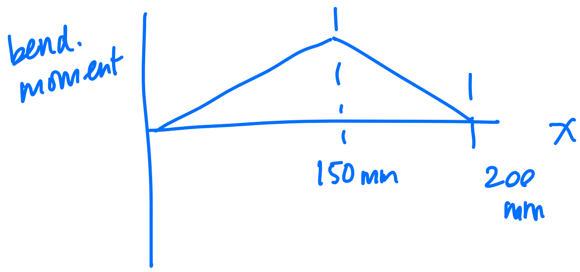


(i) A rotating shaft is supported by bearing reaction forces R_1 and R_2 and is loaded with a transverse load of 13 kN as shown in the figure.



You have three options for locating a keyseat on the rotating shaft. Which location is the best option?

- ◉ Location A
- ○ Location B
- ○ Location C



stress raisers should be located away from large bending moments → Location A will have the smallest bending.

(j) A journal bearing is designed with a nominal size of 1.25 inches.

The bushing (hole) diameter dimensions range from 1.2500 in to 1.2525 in.

The journal (shaft) diameter dimensions range from 1.2480 in to 1.2490 in.

The fit is which of the following?

Note: this question can be answered without performing any calculations.

- ◉ Close running fit
- ○ Locational transition fit
- ○ Locational interference fit
- ○ Medium drive fit
- ○ Force fit

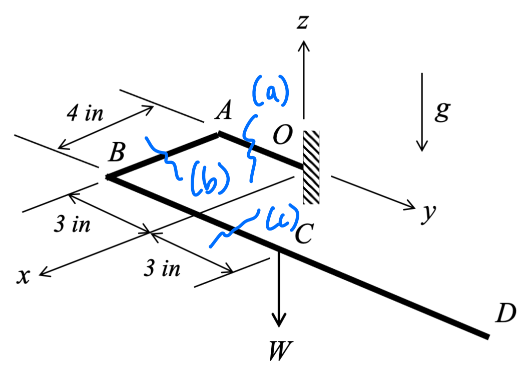
The fit described is a clearance fit; the shaft diameter is smaller than the hole diameter. The "close running fit" is the only clearance fit of the options provided.

PROBLEM No. 2 (25 points)

A paper towel holder is modeled as cantilevered beam $OABCD$. The towel holder is in the xy-plane, where gravity acts in the $-z$-direction.

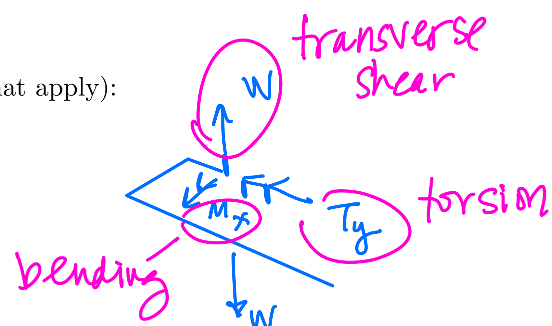
The towel holder is made of a stainless steel tube with outer diameter $D = 0.5$ inch, wall thickness $t = 0.028$ inch, cross-sectional area $A = 0.0415$ in^2, mass moment of inertia $I = 0.00116$ in^4, and polar moment of inertia $J = 0.00232$ in^4. The yield strength is $S_y = 35$ kpsi.

The weight of the tube is neglected. The weight of the paper towels is modeled as a point load W acting at location C.



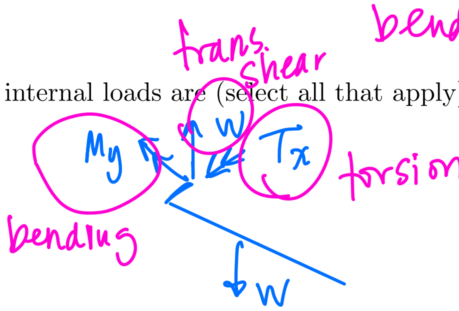
Determine the following.

(a) For a cut through segment OA, the internal loads are (select all that apply):



- ◯ Axial compression
- ◯ Axial tension
- ● Torsion
- ● Bending
- ● Transverse shear

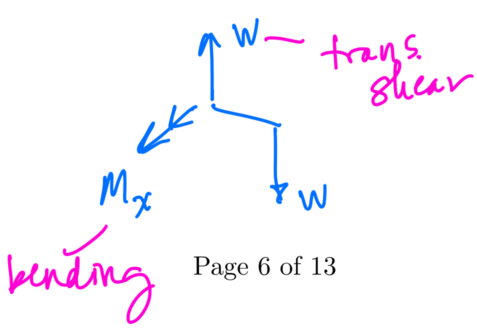
(b) For a cut through segment AB, the internal loads are (select all that apply):



- ◯ Axial compression
- ◯ Axial tension
- ● Torsion
- ● Bending
- ● Transverse shear

(c) For a cut through segment BC, the internal loads are (select all that apply):

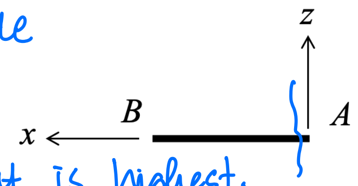
- ◯ Axial compression
- ◯ Axial tension
- ◯ Torsion
- ● Bending
- ● Transverse shear



PROBLEM No. 2 (continued)

(d) Identify the critical cross-section on segment AB. Clearly show the location on the figure below. Justify your choice.

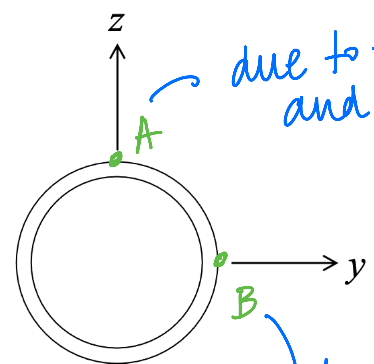
The critical cross-section will be just inside point A because the bending moment is highest.



(e) Identify the critical element(s) on the critical cross-section identified in part (d).

— Clearly show the location(s) of the critical element(s) on the sketch below.

— Justify your choice(s). You may use the attached Stress Analysis Worksheet to aid your analysis.



A: due to tensile bending stress and torsional shear stress.

B: due to transverse shear and torsional shear in the same direction

(f) For the critical element(s) identified in part (e), determine the factor of safety using the distortion energy failure theory. A roll of paper towels weighs $W = 0.6$ lbf.

$$n = \frac{S_y}{\sigma'} \qquad \sigma' = \left(\sigma_z^2 - \cancel{\sigma_x}^{0}\sigma_z + \cancel{\sigma_x}^{0\,2} + 3\tau_{xz}^2\right)^{1/2} = \frac{S_y}{n}$$

$$@A \quad \left[\left(\frac{M_y \, D/2}{I}\right)^2 + 3\left(\frac{T_x \, D/2}{J}\right)^2\right]^{1/2} = \frac{S_y}{n}$$

$$\left[\left(\frac{(4\text{in})\cdot 0.6\text{lbf}(0.5\text{in})/2}{0.00116\text{ in}^4}\right)^2 + 3\left(\frac{(6\text{in})\cdot 0.6\text{lbf}(0.5\text{in})/2}{0.00232\text{ in}^4}\right)^2\right]^{1/2} = \left(\frac{35000\text{ lbf/in}^2}{n}\right)$$

$$n = 41$$

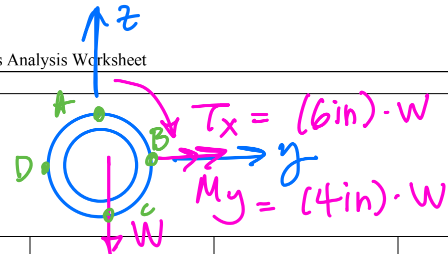
@ B $\sqrt{3}\,\tau_{xz} = \dfrac{S_y}{n}$

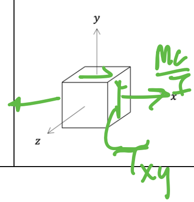
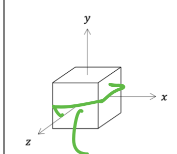
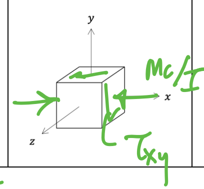
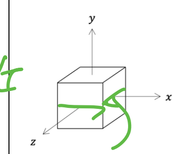
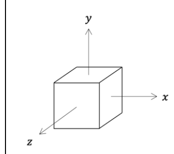
$$\sqrt{3}\left(\frac{6\text{ in}\cdot 0.6\text{ lbf}\cdot 0.5\text{ in}/2}{0.00232\text{ in}^4} + \frac{2\cdot 0.6\text{ lbf}}{0.0415\text{ in}^2}\right) = \frac{35000\text{ psi}}{n}$$

$n = 48$

→ towel holder is more likely to fail at location A on the cross-section.

- Draw the machine component's critical cross-section.
- Identify and label the potential locations for the critical element(s) (e.g., top, bottom, right, left, and center)



Potential location of critical element	A	B	C	D	
Internal load: Axial none					
Internal load: Torsion	$\tau_{xy} = +\frac{Tc}{J}$	$\tau_{xz} = -\frac{Tc}{J}$	$\tau_{xy} = -\frac{Tc}{J}$	$\tau_{xz} = +\frac{Tc}{J}$	
Internal load: Transverse shear	0	$\tau_{xz} = -\frac{2W}{A}$	0	$\tau_{xz} = -\frac{2W}{A}$	
Internal load: Bending	$\sigma_x = +\frac{Mc}{I}$	0	$\sigma_x = -\frac{Mc}{I}$	0	
Stress element					

$\tau_{xz} = \frac{Tc}{J} + \frac{2W}{A}$

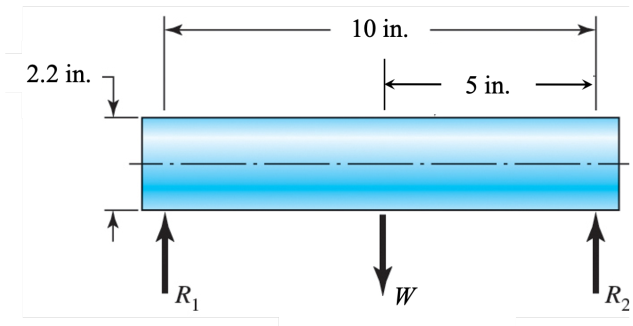
$\tau_{xz} = \frac{Tc}{J} - \frac{2W}{A}$

PROBLEM No. 3 (25 points)

A hollow shaft is made of AISI 1050 CD steel. The shaft's outer diameter is $D = 2.2$ inches and inner diameter is $d = 1.6$ inches. The cross-sectional area $A = 1.79$ in^2, mass moment of inertia $I = 0.828$ in^4, and polar moment of inertia $J = 1.656$ in^4.

The shaft rotates at a constant speed and is supported by bearing reaction forces R_1 and R_2.

The shaft operates in an environment where the temperature is 600°F. Weight W acts at the location shown in the diagram below. The desired reliability is 90%.



Determine the following.

(a) The estimated endurance limit S'_e.

(b) The fully corrected endurance limit S_e.

(c) The fatigue strength of the shaft at 1000 cycles.

(d) The maximum weight W for the rotating shaft to achieve a life of $N = 100,000$ cycles with a factor of safety of $n = 1$.

a) AISI 1050 CD steel has $S_{ut} = 100$ kpsi (Table A-20)

$S_e' = 0.5\, S_{ut} = 50$ kpsi

b) $S_e = k_a k_b k_c k_d k_e S_e'$

$k_a = a\, S_{ut}^{\,b} = 2.00\,(100)^{-0.217} = 0.736$ for CD

$k_b = 0.91\, d^{-0.157} = 0.91\,(2.2)^{-0.157} = 0.804$

$k_c = 1$ for bending

$k_d = S_T / S'_{RT} = 0.98 + 3.5 \cdot 10^{-4} \cdot 600 - 6.3 \cdot 10^{-7} \cdot 600^2 = 0.9632$

$k_e = 0.897$ for 90% reliability.

$S_e = (0.736)(0.804)(1)(0.9632)(0.897) \cdot 50 \text{ kpsi} = 25.6 \text{ kpsi}$

c) for $S_{ut} = 100$ kpsi $f = 0.844$ from Figure 6-23

$f = 1.06 - 2.8 \cdot 10^{-3} \cdot 100 + 6.9 \cdot 10^{-6} \cdot 100^2 = 0.849$ from Eqn. 6-11

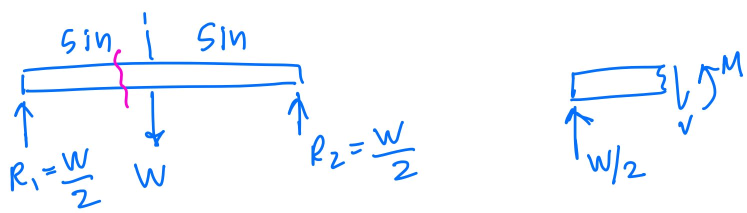
S'_f @ 1000 cycles $= f S_{ut} = 85$ kpsi

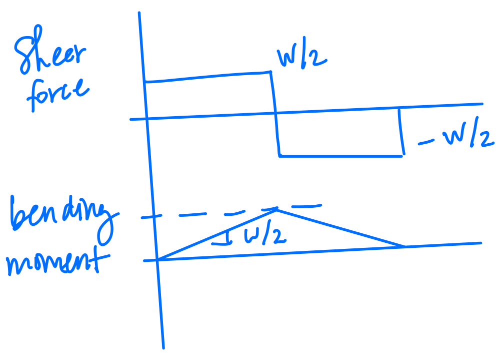
d) $S'_f = a N^b = \sigma = \dfrac{Mc}{I}$ fully reversed bending because the shaft rotates.

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{(85 \text{ kpsi})^2}{25.6 \text{ kpsi}} = 281.8 \text{ kpsi}$$

$$b = -\frac{1}{3} \log\left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3} \log\left(\frac{85 \text{ kpsi}}{25.6 \text{ kpsi}}\right) = -0.173$$

find M from a bending moment diagram.





$M_{max} = \frac{W}{2} \cdot 5 \text{ in}$

$M_{min} = -\frac{W}{2} \cdot 5 \text{ in}$

S'_f @ 100,000 cycles $= 281.8 \text{ kpsi } (100000)^{-0.173} = 38.2 \text{ Kpsi}$

$$\sigma = \frac{Mc}{I} = \frac{\frac{W}{2} \cdot 5\text{in} \cdot D_o/2}{I} = \frac{5\text{in} \cdot 2.2 \text{ in} \cdot W}{4 \cdot 0.828 \text{ in}^4} = S'_f \text{ @ } 100{,}000 \text{ cycles}$$

$$W = \frac{38.2 \text{ kpsi} \cdot 4 \cdot 0.828 \text{ in}^4}{5\text{in} \cdot 2.2 \text{ in}} = 11.5 \text{ kips}$$

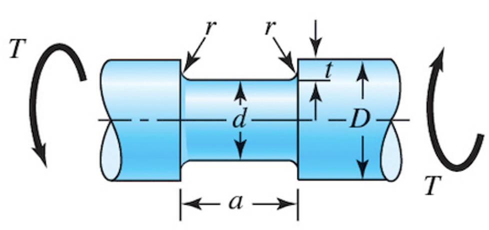
PROBLEM No. 4 (25 points)

The rotating round shaft with a flat-bottom groove is loaded with a torque T that varies between T_{min} and T_{max}, where T_a and T_m are related by:

$$\frac{T_a}{T_m} = 0.5$$

The shaft's dimensions are $r = 1.0$ mm, $t = 10$ mm, $a = 25$ mm, $d = 20$ mm, and $D = 40$ mm.

The shaft has ultimate tensile strength $S_{ut} = 700$ MPa and yield strength $S_y = 590$ MPa. The fully corrected endurance limit is $S_e = 170$ MPa.



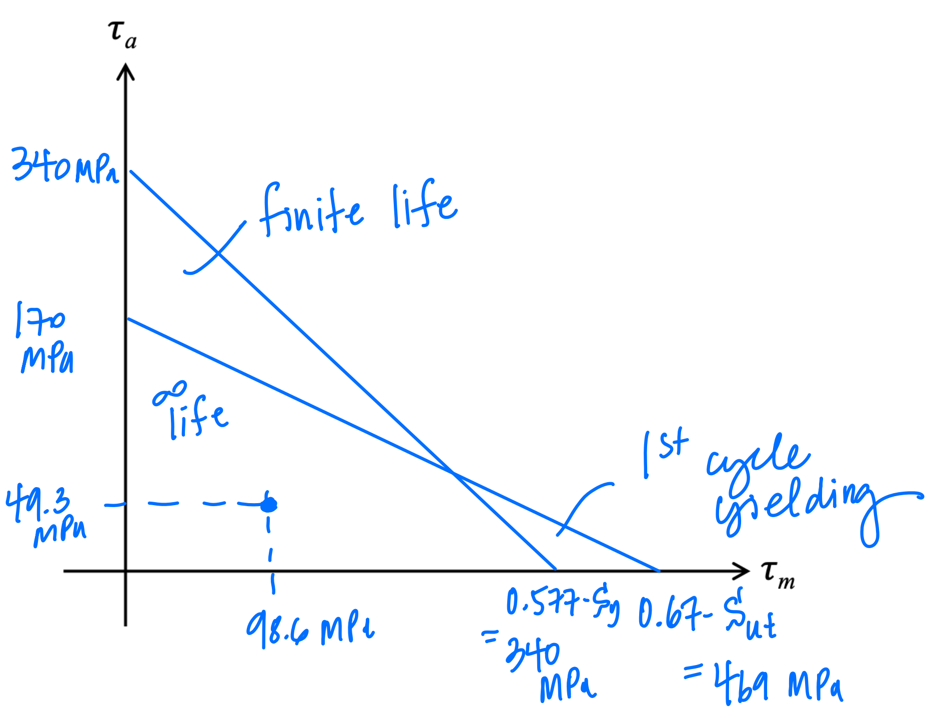
Determine the following.

(a) The fatigue stress concentration factor K_{fs}.

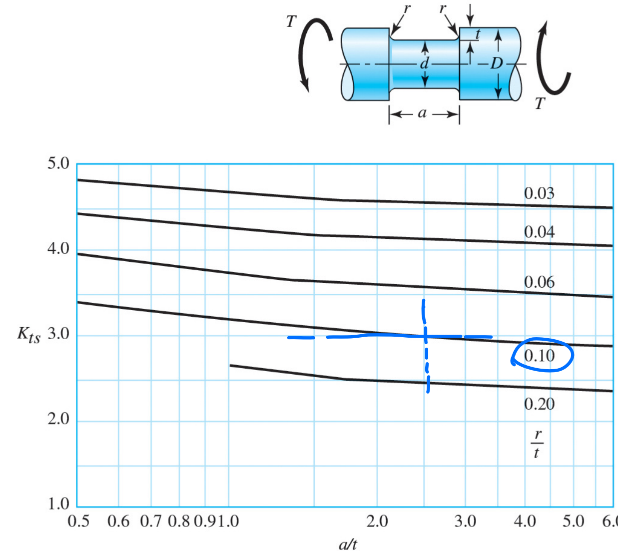
(b) T_{min} and T_{max} for the rotating shaft such that the factor of safety for infinite life found with the Goodman criterion is $n_f = 2$.

(c) Check for first-cycle yielding.

(d) Sketch and label the stress state on a fluctuating-stress diagram using the axes provided. Show the Goodman line, the yield line, and the zones for infinite life, finite life, and first-cycle yielding.



PROBLEM No. 4 (continued)



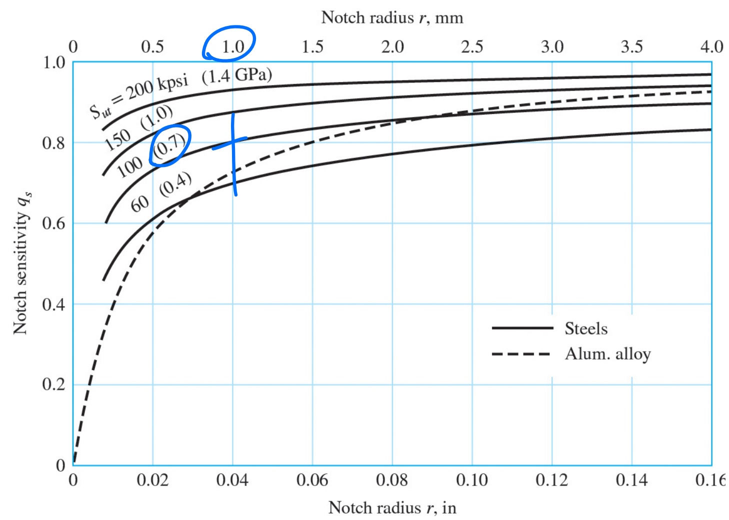
a) $K_{fs} = 1 + q_s(K_{ts} - 1)$

Find K_{ts} from Figure A-15-17

$\frac{r}{t} = \frac{1\ mm}{10\ mm} = 0.1$

$\frac{a}{t} = \frac{25\ mm}{10\ mm} = 2.5$

$K_{ts} = 3.0$



q_s from Figure 6-27

$q_s = 0.8$ for $r = 1.0$ mm, $S_{ut} = 700$ MPa

$K_{fs} = 1 + 0.8(3-1) = 2.6$

b) $$\frac{1}{n_f} = \frac{\cancel{\sigma_a}\ \tau_a}{S_e} + \frac{\cancel{\sigma_m}\ \tau_m}{\cancel{S_{ut}}\ 0.67\, S'_{ut}}$$

fully corrected S_e given includes k_c for torsion.

$\tau_o = \frac{16T}{\pi d^3}$ from caption of Figure A-15-17

$$\tau_a = K_{fs} \cdot \frac{16 T_a}{\pi d^3} = K_{fs} \cdot \frac{16 \cdot 0.5 T_m}{\pi d^3} = 2.6 \cdot \frac{16 \cdot 0.5\, T_m}{\pi (0.02 m)^3}$$

$$\tau_m = K_{fs} \cdot \frac{16 T_m}{\pi d^3} = 2 \tau_a$$

$$\frac{1}{2} = \frac{\tau_a}{170 \text{ MPa}} + \frac{2\tau_a}{0.67 \cdot 700 \text{ MPa}} = \tau_a \left(\frac{1}{170} + \frac{2}{0.67 \cdot 700} \right)$$

$\tau_a = 49.3$ MPa $\quad \tau_m = 2\tau_a = 98.6$ MPa

$$T_a = \frac{\tau_a}{K_{fs}} \cdot \frac{\pi d^3}{16} = \frac{49.3 \cdot 10^6 \text{Pa}}{2.6} \cdot \frac{\pi \cdot (0.02 \text{ m})^3}{16} = 29.8 \text{ N-m}$$

$T_m = 2 T_a = 59.5$ N-m

$T_{max} = T_m + T_a = 89.4$ N-m

$T_{min} = T_m - T_a = 29.8$ N-m

c) $$n = \frac{S_{sy}}{\tau_a + \tau_m} = \frac{0.577\, S_y}{\tau_a + \tau_m} = \frac{0.577 \cdot 590 \text{ MPa}}{49.3 \text{MPa} + 2 \cdot 49.3 \text{ MPa}} = 2.3$$