Problem 1 (xx points):

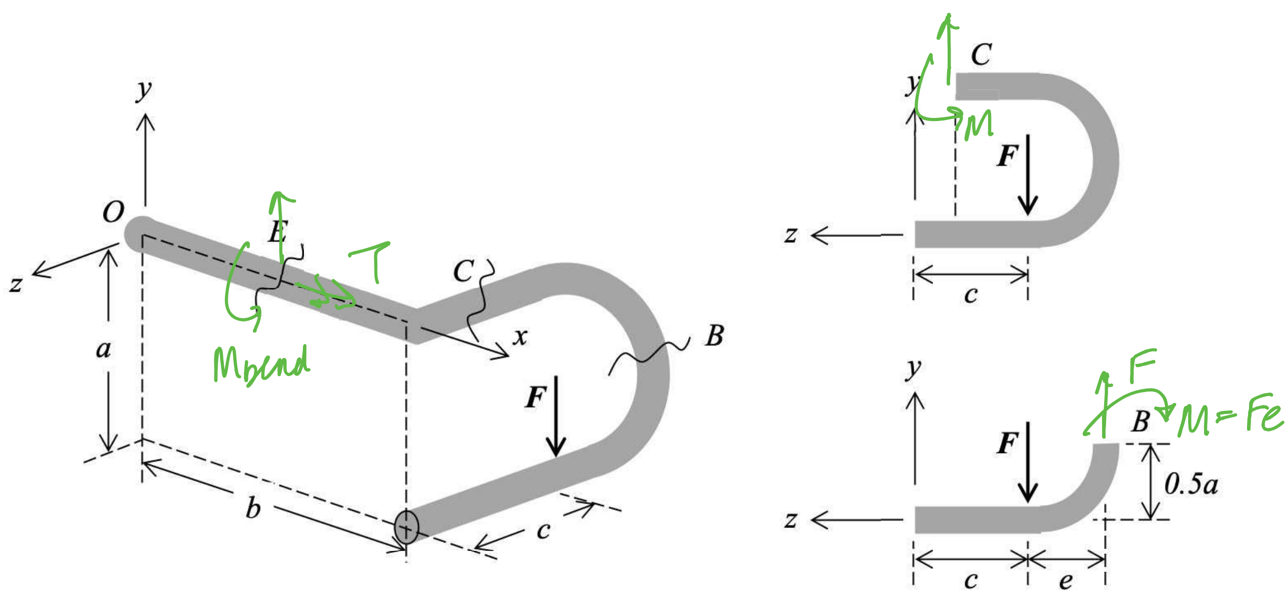
Given:. Drop handlebars have steered road bikes since the late 1800s.

A schematic of one half of a set of drop handlebars is shown below.

The handlebars are made of a hollow tube, with outer diameter $D = 1$ inch, inner diameter $d = 0.81$ inch, cross-sectional area $A = 0.27$ in^2, $I = 0.0280$ in^4, and $J = 0.0560$ in^4.

The handlebars are clamped at O.

Force $\vec{F} = -50\vec{j}$ lbf acts at the location shown. The handlebar dimensions are $a = 7$ inches, $b = 10$ inches, $c = 6$ inches, and $e = 3$ inches.



(i) For a cut through the handlebars at location B, the internal loads are (select all that apply):

- ○ Axial compression
- ● Axial tension
- ○ Torsion
- ● Bending
- ○ Transverse shear

(ii) For a cut through the handlebars at location C, the internal loads are (select all that apply):

- ○ Axial compression
- ○ Axial tension
- ○ Torsion
- ● Bending
- ● Transverse shear

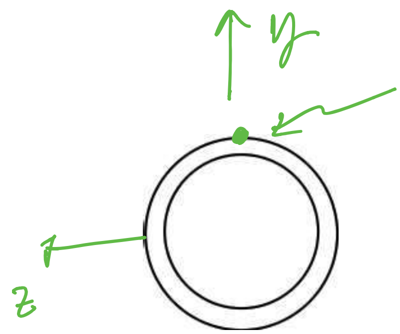
(iii) For a cut through the handlebars at location E, the internal loads are (select all that apply):

- ○ Axial compression
- ○ Axial tension
- ● Torsion
- ● Bending
- ● Transverse shear

(iv) Identify the critical cross-section of handlebars. Justify your choice.

(v) Identify the critical element on the critical cross-section.

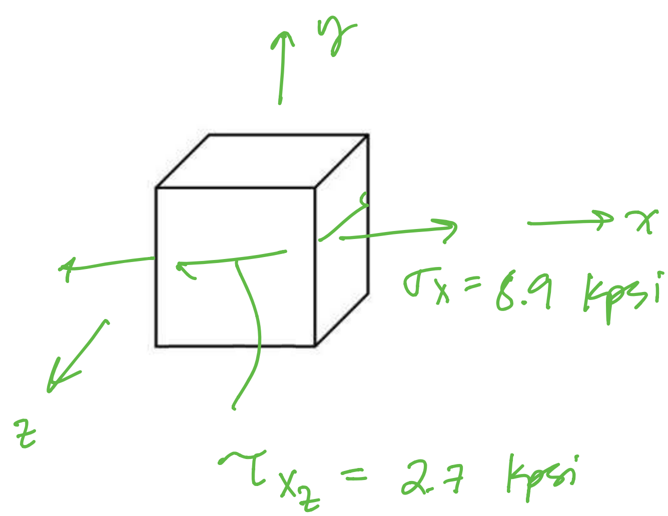
— Clearly show the location of the critical element on the sketch below.
— Clearly label the x-, y-, and/or z-axes.
— You may use the attached Stress Analysis Worksheet to aid your analysis.



critical element due to tensile bending stress and torsional shear stress

(vi) Draw the stress state for the critical element identified.

— Include the magnitude of each stress component.
— Clearly label the x-, y-, and z-axes.



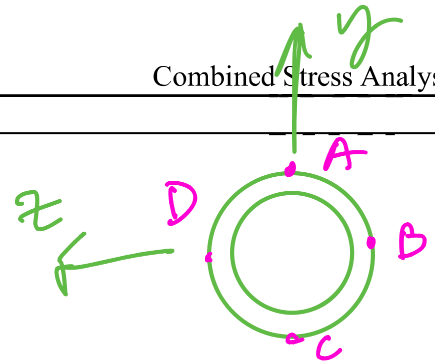
iv) the critical cross-section is at the wall because that is the location with the highest bending moment.

v) see attached.

vi) $$\sigma_x = \frac{Mc}{I} = \frac{|F| \cdot b \cdot D/2}{I} = \frac{50 \text{ lbf} \cdot 10 \text{ in} \cdot 1 \text{ in}/2}{0.028 \text{ in}^4} = 8.9 \text{ kpsi}$$

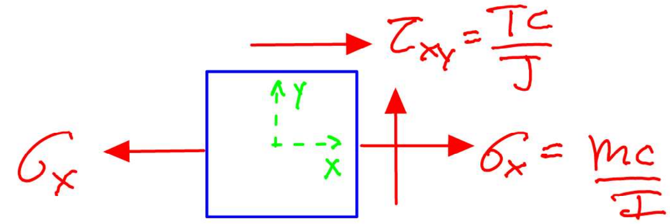
$$\tau_{xz} = \frac{Tc}{J} = \frac{|F| \cdot c \cdot D/2}{J} = \frac{50 \text{ lbf} \cdot 6 \text{ in} \cdot 1 \text{ in}/2}{0.056 \text{ in}^4} = 2.7 \text{ kpsi}$$

- Draw the machine component's critical cross-section.
- Identify and label the potential locations for the critical element(s) (e.g., top, bottom, right, left, and center)



		A	B	C	D	
Potential location of critical element		A	B	C	D	
Internal load	Axial: none					
	Torsion	$\tau_{xz} = -\frac{Tc}{J}$	$\tau_{xy} = -\frac{Tc}{J}$	$\tau_{xz} = +\frac{Tc}{J}$	$\tau_{xy} = +\frac{Tc}{J}$	
	Transverse shear	0	$\tau_{xy} = -2\frac{V}{A}$	0	$\tau_{xy} = -2\frac{V}{A}$	
	Bending	$\sigma_x = +\frac{Mc}{I}$	0	$\sigma_x = -\frac{Mc}{I}$	0	
Stress element		$\frac{Mc}{I}$ $\tau = \frac{Tc}{J}$ critical element	$2\frac{V}{A} + \frac{Tc}{J}$	$\frac{Mc}{I}$ $\frac{Tc}{J}$	$\frac{Tc}{J} - 2\frac{V}{A}$	

Problem 2



(A.) (15 pts)

$$\sigma_x = \frac{Mc}{I} = \frac{(75\,N\cdot m)(d/2)}{(\pi d^4/64)} = \frac{763.94}{d^3}$$ 3 pts

$$\tau_{xy} = \frac{Tc}{J} = \frac{(45\,N\cdot m)(d/2)}{(\pi d^4/32)} = \frac{229.18}{d^3}$$ 3 pts

(i) For M.S.S. theory: $\eta = \frac{S_y}{2\tau_{max}} \Rightarrow \tau_{max} = \frac{S_y}{2\eta}$ (eq 5-3)

For $\eta = 2.5$, $\tau_{max} = \frac{S_y}{2(2.5)} = \frac{180\times10^6\,Pa}{5} = 36\times10^6\,Pa$ ① 3 pts

and $\tau_{max} = \sqrt{\left(\frac{\sigma_x - \cancel{\sigma_y}^{\,0}}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{763.94}{2d^3}\right)^2 + \left(\frac{229.18}{d^3}\right)^2}$ (Fig 3-10 Mohr's circle)

$$= \sqrt{\left(\frac{381.97}{d^3}\right)^2 + \left(\frac{229.18}{d^3}\right)^2} = \frac{445.45}{d^3}$$ ② 3 pts

① = ② $\quad 36\times10^6 = \frac{445.45}{d^3} \Rightarrow d^3 = \frac{445.45}{36\times10^6}$

$\therefore d = 0.0231\ m \Rightarrow \underline{d = 23.1\ mm}$ 1 pt

(ii)

D.E. should have a smaller value for d since M.S.S. is more conservative failure theory.

1 pt.

1 pt.

$$\therefore \quad d_{D.E.} < d_{M.S.S.}$$

---

* check *

For D.E. theory $\quad \eta = \dfrac{S_y}{\sigma'} \Rightarrow \sigma' = \dfrac{S_y}{\eta} = \dfrac{180\times10^6}{2.5} = 72\times10^6 \text{ Pa} \quad (3)$

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} = \sqrt{\left(\frac{763.94}{d^3}\right)^2 + 3\left(\frac{229.18}{d^3}\right)^2} = \frac{860.915}{d^3} \quad (4)$$

(3) = (4)

$$72\times10^6 = \frac{860.915}{d^3} \Rightarrow d^3 = \frac{860.915}{72\times10^6}$$

$$\therefore d = 0.02286 \text{ m} \Rightarrow \underline{d = 22.9 \text{ mm}}$$

$$\therefore d_{DE} = 22.9 \text{ mm} < d_{MSS} = 23.1 \text{ mm} \checkmark$$

(B) ASTM cast iron: $S_{UT} = 214$ MPa , $S_{UC} = 752$ MPa

(15 pts) $d = 35$ mm

(i) $$\sigma_x = \frac{763.94}{d^3} = \frac{763.94}{(0.035\,m)^3} = 17.82 \text{ MPa}$$ ✓ 2 pts

$$\tau_{xy} = \frac{229.18}{d^3} = \frac{229.18}{(0.035\,m)^3} = 5.35 \text{ MPa}$$ ✓ 2 pts

(ii)

BCM and MM theories require values for σ_A and σ_B

$$\sigma_A, \sigma_B = \sigma_1, \sigma_2 = \frac{\sigma_x + \cancelto{0}{\sigma_y}}{2} \pm \sqrt{\left(\frac{\sigma_x^2 - \cancelto{0}{\sigma_y}}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{17.82}{2} \pm \sqrt{\left(\frac{17.82}{2}\right)^2 + (5.35)^2}$$

$$= 8.91 \pm 10.39$$

$\therefore$ $\sigma_A = 19.3$ MPa (2 pts) $\sigma_B = -1.48$ MPa (2 pts)

For B.C.M. Theory:

eq 5-31b for $\sigma_A \geq 0 \geq \sigma_B$ 5 pts.

$$n = \left(\frac{\sigma_A}{S_{UT}} - \frac{\sigma_B}{S_{UC}}\right)^{-1} = \left(\frac{19.3}{214} - \frac{(-1.48)}{752}\right)^{-1} = \underline{10.85}$$

(iii)

Modified Mohr theory is less conservative theory so expect that $\eta_{MM} > \eta_{BCM}$ ✓

1 pt.

1 pt.

---

* check *

For M.M. Theory:

eq 5-32b for $\sigma_A \geq 0 \geq \sigma_B$

$$\eta = \left[\frac{(S'_{UC} - S'_{UT})\sigma_A}{S'_{UC} S'_{UT}} - \frac{\sigma_B}{S'_{UC}}\right]^{-1} = \left[\frac{(752-214)(19.3)}{(752)(214)} - \frac{(-1.48)}{752}\right]^{-1}$$

$$\eta = 15.04$$

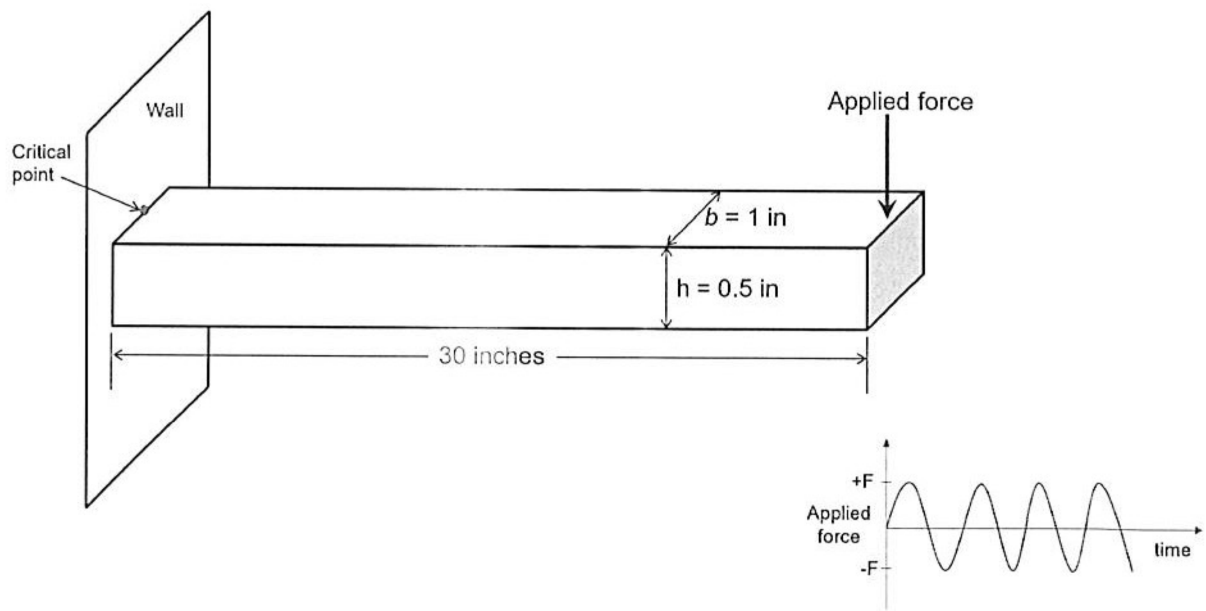
$\therefore \; \eta_{mm} = 15.04 > \eta_{BCM} = 10.85$ ✓

SOLUTION

Problem 3 (30 points):

A solid rod with a rectangular cross-section ($h = 0.5$ inch, $b = 1.0$ inch) is cantilevered at one end. The rod is 30 inches long, and supports a completely reversed transverse load at the other end of $\pm F$ lbf. The material is AISI 1045 hot-rolled steel. The rod operates at room temperature, and has a reliability of 99.99%. Neglecting any stress concentration, calculate the following for the critical point on the critical section at the wall:

4 (i) the uncorrected endurance strength, S_e'.
10 (ii) the fully corrected endurance strength of the rod, S_e.
2 (iii) the fatigue strength of the rod for 10^3 cycles.
6 (iv) the fatigue strength of the rod for 50,000 cycles, S_f.
5 (v) the life of the rod if the amplitude of the force is $F = 30$ lbf.
3 (vi) the maximum amplitude of the force F that would result in infinite life of the rod.



$h = 0.5''$
$b = 1.0''$

$l = 30''$

(2) points

AISI 1045 HR steel

$S_{ut} = 82$ kpsi
$S_y = 45$ kpsi
pg. 1056 (table A-20)

Room temp
$R = 99.99\%$.

(ii) $S_e' = \frac{1}{2} 82$ kpsi

(2) points

$\boxed{S_e' = 41 \text{ kpsi}}$

(i) $K_a = 11.0\,(82)^{-0.650}$ for Hot rolled (Table 6-2 pg 311)

(2) points

$\boxed{K_a = 0.6272}$

$K_b = 0.879\, d_e^{-0.107}$

where $d_e = 0.808\sqrt{hb}$
$= 0.5713$

(2) points

$K_b = 0.879\,(0.5713)^{-0.107}$

(2) points

$\boxed{K_b = 0.9333}$

$K_c = 1$

$K_d = 1$

$\boxed{K_e = 0.702}$ for $R = 99.99\%$

(2) points

$\Rightarrow S_e = (0.6272)(0.9333)(0.702)(41)$

$\boxed{S_e = 16.848}$ kpsi

(2) points

(iii) $S_m = f S_{ut}$

$$= \left(1.06 - 2.8\times10^{-3}(82) + 6.9\times(10^{-6})(82)^2\right)(82)$$

$$= (0.8768)(82)$$

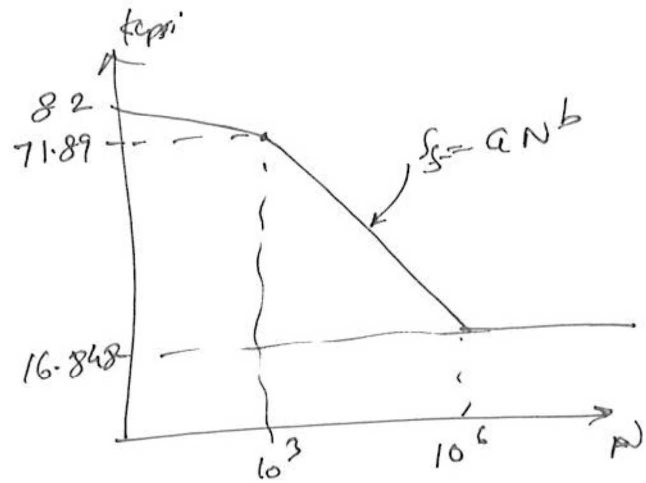
$$\boxed{S_m = 71.89 \text{ Kpsi}}$$

2 points

(iv)

$$a = \frac{(f S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right)$$



where $a = \dfrac{(71.89)^2}{16.848} = 306.753$ Kpsi

2 points

$$b = -\frac{1}{3}\log\left(\frac{71.89}{16.848}\right) = -0.21$$

2 points

$$\therefore \quad S_f = 306.753\, N^{-0.21}$$

For $N = 50{,}000$, $S_f = 306.753(50{,}000)^{-0.21}$

$$\boxed{S_f = 31.62 \text{ Kpsi}}$$ @ 50,000 revs

2 points

(v) $\sigma = \dfrac{M\left(\frac{h}{2}\right)}{I} = \dfrac{M\left(\frac{h}{2}\right)}{\frac{1}{12} b h^3} = \dfrac{6M}{bh^2} = \dfrac{6Fl}{bh^2}$

$= \dfrac{6 \times 30 \times 30''}{(1)(0.5)^2}$

$\boxed{\sigma = 21.6 \text{ kpsi}}$ — (2) points

$N = \left(\dfrac{\sigma_{ar}}{a}\right)^{1/b} = \left(\dfrac{21.6 \text{ kpsi}}{306.75 \text{ kpsi}}\right)^{-1/0.21}$

$\boxed{N = 3.0712 \times 10^5 \text{ cycles}}$ — (3) points

(vi) for infinite life,

$\sigma = S_e$

$\Rightarrow \dfrac{6Fl}{bh^2} = 16.848 \times 10^3 \text{ psi}$

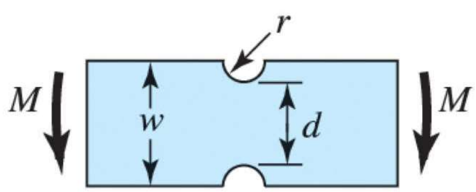
$\Rightarrow F = 16848 \times \left(\dfrac{bh^2}{6l}\right) = 16848 \times \dfrac{1 \times (0.5)^2}{6 \times 30}$

$\boxed{F = 23.4 \text{ lbf}}$ — (3) points

Problem 4 (15 points):

Given: A notched rectangular bar is made of AISI 1035 CD steel. The bar has dimensions $w = 1.5$ inches and $d = 1$ inch. The bar's thickness (the dimension into the page) is 0.5 inch.

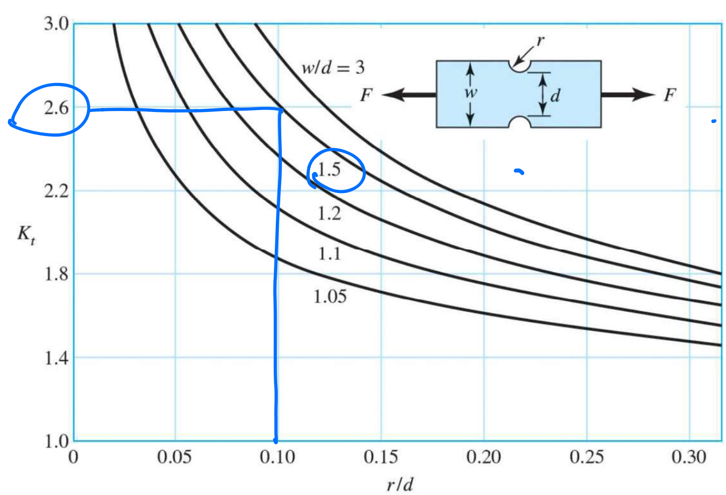
When loaded in tension or simple compression, the bar's stress concentration factor is $K_t = 2.6$.



Determine the following for the notched rectangular bar loaded in bending.

(i) The notch radius, r.
(ii) The fatigue stress concentration factor K_f at the notch.
(iii) The stress at the notch if $M = 50$ in-lbf.

i) find r from given geometry and K_t for axial

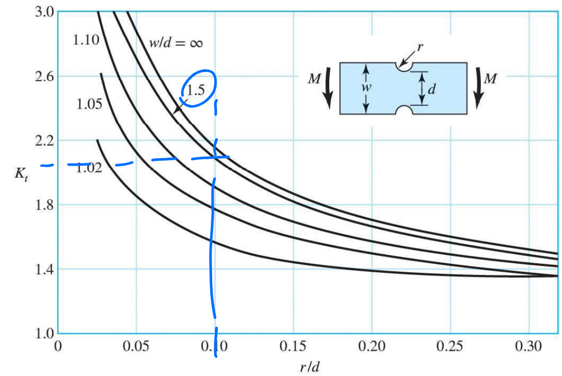


$\frac{w}{d} = \frac{1.5 in}{1 in} = 1.5$

from Fig A-15-3, $\frac{r}{d} = 1$

$\rightarrow r = 0.1$ in

ii) from Fig. A-15-4 with $\frac{r}{d} = 0.1$ and $\frac{w}{d} = 1.5$ $\quad K_t = 2.1$

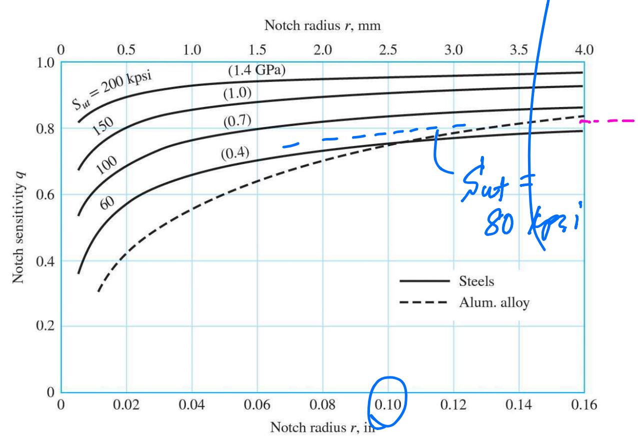


$$K_f = 1 + q(K_t - 1)$$

Find q from Figure 6-26

$S_{ut} = 80$ kpsi for AISI 1035 CD from Table A-20

pick $q = 0.8$



Source: Sines, George and Waisman, J. L. (eds.), Metal Fatigue, McGraw-Hill, New York, 1969.

$$K_f = 1 + 0.8(2.1 - 1)$$
$$= 1.88$$

q could also be calculated from eqn. 6-35 and 6-33

$$\sqrt{a} = 0.246 - 3.08(10^{-3}) \cdot 80 + 1.51(10^{-5}) \cdot 80^2 - 2.67(10^{-8}) \cdot 80^3$$
$$= 0.0825696$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.0825696}{\sqrt{0.1}}} = 0.793$$

very close to estimate from Fig 6-26

iii) from caption of Fig A-15-3

$$\sigma_0 = \frac{Mc}{I} = \frac{M\, d/2}{t d^3/12}$$

$$\sigma = K_f \sigma_0 = 1.88 \cdot \frac{50\ \text{in-lbf} \cdot (1\ \text{in}/2)}{(0.5\ \text{in})(1\ \text{in})^3/12} = 1128\ \text{psi}$$