 Name: solution

April 6, 2022

INSTRUCTIONS

Begin each problem in the space provided. If additional space is required, use the paper provided to you.

Work appearing on the backside of any exam page will NOT be graded.

If your solution does not follow a logical thought process, it will be assumed to be in error.

PROBLEM No. 1 (25 points)

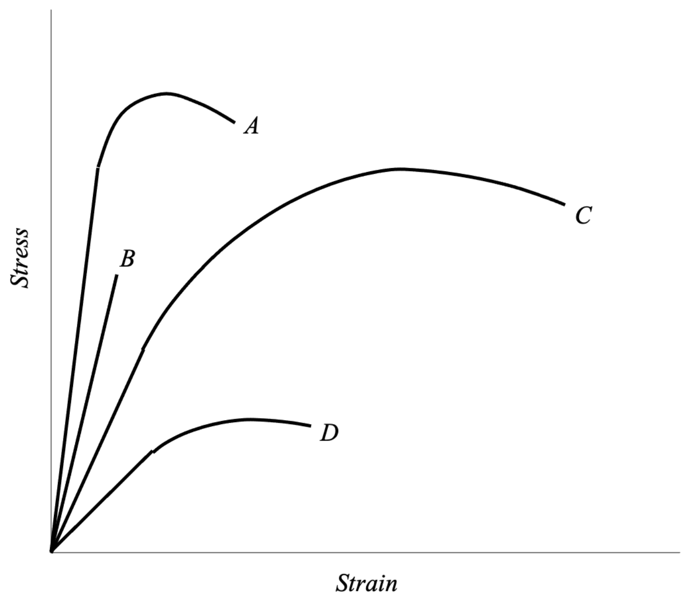
Problem 1 consists of 10 questions. Each question is worth 2.5 points.

(a) SAE 1060 steel is a (select one of the following):

- ○ Low-carbon steel
- ○ Medium-carbon steel
- ● High-carbon steel
- ○ Ultra-high-carbon steel

60 in 1060 means 0.6 % carbon by weight → high-carbon

(b) Stress-strain curves are shown below for four materials.



The most ductile material is (select one of the following):

- ○ Material A
- ○ Material B
- ● Material C
- ○ Material D

The strongest material is (select one of the following):

- ● Material A
- ○ Material B
- ○ Material C
- ○ Material D

The best material for a spring is (select one of the following):

- ○ Material A
- ○ Material B
- ● Material C
- ○ Material D

largest area under the curve.

(c) Spokes on a bicycle wheel are most likely which of the following?

○ Hot rolled

● Cold worked

In a few words, justify your answer.

cold worked parts have bright, new finish and are more accurate

(d) Which do you think is fastest?

○ Roller skating on asphalt

○ Rollerblading on asphalt

○ Ice skating

● It depends

Roller skates | Rollerblades | Ice skates

In a few words, justify your answer.

a reasonable justification would include some discussion of rolling vs. sliding contact, friction and/or # of parts

(e) The stiffness of a torsion spring can be decreased by which of the following? Select all that apply.

☐ Increasing the wire diameter d $d\uparrow \rightarrow k'\uparrow$

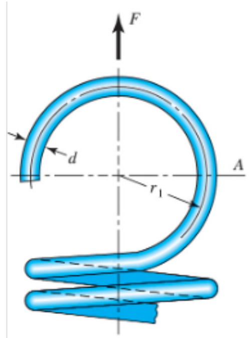
■ Increasing the number of body coils $N_b\uparrow \rightarrow N_a\uparrow \rightarrow k'\downarrow$

■ Increasing the length of the spring ends $l_1\uparrow \rightarrow N_a\uparrow \rightarrow k'\downarrow$

■ Changing the material from chrome-silicon A401 to HD spring A227 $E\downarrow \rightarrow k'\downarrow$

$$k' = \frac{d^4 E}{10.8\, D N_a} \qquad N_a = N_b + \frac{l_1 + l_2}{3\pi D}$$

(f) The music wire used in a helical extension spring has ultimate tensile strength $S_{ut} = 275$ kpsi.



The yield strength at location A in the spring hook is most likely:

- ○ 165 kpsi
- ● 205 kpsi
- ○ 220 kpsi
- ○ 240 kpsi
- ○ 275 kpsi

Table 10-5: S_y is 65-75% of S_{ut}
→ between 179 and 206 kpsi
Table 10-7: S_y is 75% of S_{ut} = 206 kpsi

(g) The most appropriate lubrication type when velocities are small and when friction is to be an absolute minimum is:

- ○ Boundary
- ○ Mixed-film
- ○ Hydrodynamic
- ● Hydrostatic → complete separation between components
- ○ Elastohydrodynamic

(h) The type of lubrication when rolling element bearings are lubricated is:

- ○ Boundary
- ○ Mixed-film
- ○ Hydrodynamic
- ○ Hydrostatic
- ● Elastohydrodynamic

(i) Catalog data are shown below for a deep groove ball bearing. The catalog rating life is 10^6 cycles. Determine the radial load that can be supported for 10^7 cycles with 90% reliability.

Principal dimensions			Basic load ratings dynamic	static	Fatigue load limit	Speed ratings Reference speed	Limiting speed1)	Mass	Designations Bearing open or capped on both sides	capped on one side1)
d	D	B	C	C_0	P_u					
mm			kN		kN	r/min		kg	–	
12	21	5	1,74	0,915	0,039	–	20 000	0,0063	▸ **61801-2RS1**	–

- ◯ 425 N
- ◯ 460 N
- ● 800 N
- ◯ 850 N
- ◯ 915 N
- ◯ 1740 N
- ◯ None of the above
- ◯ Cannot be determined from the given information

$$a_1 F_R L_R^{1/a} = F_D L_D^{1/a}$$

$$F_D = \frac{1 \cdot 1.74 \cdot (10^6)^{1/3}}{(10^7)^{1/3}} = 0.808 \text{ kN}$$

(j) Two spheres, each with Poisson's ratio $\nu = 0.3$, are pressed together with a 10 N force.

The radius of the circular contact area is $a = 0.0990$ mm.

Determine the maximum shear stress and the depth at which the the maximum shear stress occurs in the spheres.

$$p_{max} = \frac{3F}{2\pi a^2} = \frac{3 \cdot 10\text{ N}}{2\pi\left(0.099\text{ mm} \cdot \frac{1\text{m}}{1000\text{mm}}\right)^2} = 490 \text{ MPa}$$

$$\tau_{max} = 0.3\, p_{max} = 146 \text{ MPa}$$

$$z = 0.48 \cdot 0.099 \text{ mm} = 0.04752 \text{ mm}$$

PROBLEM No. 2 (16 points)

A part is loaded with a combination of bending, axial, and torsion such that the following nominal stresses are created at a particular location.

Bending: Completely reversed, with a maximum stress of 60 MPa

Axial: Constant stress of 20 MPa

Torsion: Repeated load, varying from 0 MPa to 50 MPa

Assume the varying stresses are in phase with each other.

The part contains a notch such that $K_{f,bending} = 1.4$, $K_{f,axial} = 1.1$, and $K_{fs,torsion} = 2.0$.

The material properties are $S_y = 300$ MPa and $S_{ut} = 400$ MPa.

The completely adjusted endurance limit is $S_e = 200$ MPa.

Determine the following.

(a) The factor of safety for infinite life using the Goodman criterion. If infinite life is not predicted, estimate the number of cycles to failure.

(b) Check for first-cycle yielding.

a) $$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$$

$$\sigma_a' = \left\{ \left[(K_f)_{bending} \sigma_{ao,bending} + (K_f)_{axial} \cancel{\sigma_{ao,axial}} \right]^2 + 3\left[(K_{fs})_{torsion} \tau_{ao,torsion} \right]^2 \right\}^{1/2}$$

$$\sigma' = \left\{ \left[(K_f)_{bending} \cancelto{0}{\sigma_{mo,bending}} + (K_f)_{axial} \sigma_{mo,axial} \right]^2 + 3\left[(K_{fs})_{torsion} \tau_{mo,torsion} \right]^2 \right\}^{1/2}$$

Bending completely reversed

$\sigma_{ao,bending} = 60$ MPa

$\sigma_{mo,bending} = 0$

PROBLEM No. 2 (continued)

Axial constant

$\sigma_{ao,\,axial} = 0$

$\sigma_{mo,\,axial} = 20 \text{ MPa}$

Torsion repeated

$\tau_{ao,\,torsion} = 25 \text{ MPa}$

$\tau_{mo,\,torsion} = 25 \text{ MPa}$

$$\sigma_a' = \left\{ [1.4 \cdot 60 \text{ MPa}]^2 + 3[2.0 \cdot 25 \text{ MPa}]^2 \right\}^{1/2} = 120.6 \text{ MPa}$$

$$\sigma_m' = \left\{ [1.1 \cdot 20 \text{ MPa}]^2 + 3[2.0 \cdot 25 \text{ MPa}]^2 \right\}^{1/2} = 89.4 \text{ MPa}$$

$$\frac{1}{n_f} = \frac{120.6 \text{ MPa}}{200 \text{ MPa}} + \frac{89.4 \text{ MPa}}{400 \text{ MPa}} \rightarrow n_f = 1.2$$

infinite life is predicted.

b) $$n \approx \frac{S_y}{\sigma_a' + \sigma_m'} = \frac{300 \text{ MPa}}{(120.6 + 89.4) \text{ MPa}} = 1.4$$

PROBLEM No. 3 (25 points)

A helical compression spring is subjected to cyclic loading, where the minimum force is 60 lbf and the maximum force is 150 lbf over a deflection of 1 inch.

The spring index is $C = 7$ and the wire diameter is $d = 0.207$ in. The spring is made of music wire and the spring has been peened. The spring ends are squared and ground.

Determine the following.

(a) The factor of safety for infinite life using the Goodman criterion.

(b) The spring rate.

(c) The total number of coils.

$$\frac{1}{n_f} = \frac{\cancel{\sigma_a} \to \tau_a}{\cancel{S_e} \to S'_{se}} + \frac{\cancel{\sigma_m} \to \tau_m}{\cancel{S_{ut}} \to S'_{su}}$$

$$\tau_{min} = K_B \frac{8 F_{min} D}{\pi d^3} = 1.2 \cdot \frac{8 \cdot 60\text{ lbf} \cdot 1.449\text{ in}}{\pi \cdot (0.207\text{ in})^3} = 30\text{ Kpsi}$$

$$K_B = \frac{4C+2}{4C-3} = \frac{4 \cdot 7 + 2}{4 \cdot 7 - 3} = \frac{30}{25} = 1.2$$

$$F_{min} = 60\text{ lbf}$$

$$C = \frac{D}{d} \rightarrow D = Cd = 7 \cdot 0.207\text{ in} = 1.449\text{ in}$$

$$\tau_{max} = K_B \frac{8 F_{max} D}{\pi d^3} = 74.9\text{ Kpsi}$$

$$F_{max} = 150\text{ lbf}$$

PROBLEM No. 3 (continued)

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = 22.5 \text{ kpsi}$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = 52.4 \text{ kpsi}$$

$$S_{se} = \frac{S_{sa}}{1 - \frac{S_{sm}}{S_{su}}} = \frac{57.5 \text{ kpsi}}{1 - \frac{77.5 \text{ kpsi}}{169.2 \text{ kpsi}}} = 106 \text{ kpsi}$$

$$S_{su} = 0.67\, S_{ut} = 169.2 \text{ kpsi}$$

$$S_{ut} = \frac{A}{d^m} = \frac{201}{0.207^{0.145}} = 252.5 \text{ kpsi}$$

$$n_f = \left(\frac{22.5 \text{ kpsi}}{106. \text{ kpsi}} + \frac{52.4 \text{ kpsi}}{169.2 \text{ kpsi}} \right)^{-1} = 1.9$$

b) $k = \frac{F}{y} = \frac{150 \text{ lbf} - 60 \text{ lbf}}{1 \text{ in}} = 90 \text{ lbf/in}$

c) $k \approx \frac{d^4 G}{8 D^3 N_a} = \frac{(0.207 \text{ in})^4 \cdot 11.6 \cdot 10^6 \text{ lbf/in}^2}{8 \cdot (1.449 \text{ in})^3 \cdot N_a} = 90 \text{ lbf/in}$

$N_a = 9.7$ coils

$N_t = N_a + 2 = 11.7$ coils

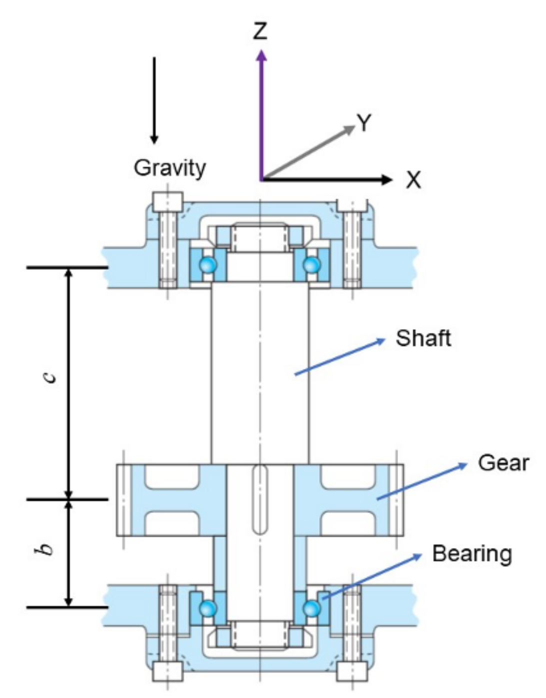
PROBLEM No. 4 (16 points)

Two bearings support a shaft and gear. The shaft and gear have total mass 133 kg. The shaft rotates at 30 rpm.

The load acting on the gear is $\vec{F} = 3\vec{i} + 4\vec{j}$ kN.

The distance between the gear and the bottom bearing is $b = 20$ cm. The distance between the gear and the top bearing is $c = 50$ cm.

The angular contact ball bearing selected for the bottom bearing has catalog rating loads $C_{10} = 14.8$ kN and $C_0 = 7.65$ kN.



Determine the following.

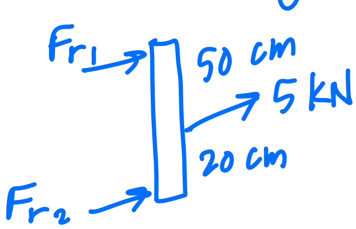
(a) The axial load acting at the bottom bearing.

(b) The radial load acting at the bottom bearing.

(c) The life of the bottom bearing if 95% reliability is required.

a) $F_a = 133 kg \cdot 9.81\ m/s^2 = 1.3\ kN$

b)



$F_{r2} = 50\ cm \cdot 5\ kN / 70\ cm = 357\ kN$

PROBLEM No. 4 (continued)

c) $a_1 F_R L_R^{1/a} = F_D L_D^{1/a}$

$a_1 = 0.64$ for 95% reliability

$F_R = C_{10} = 14.8\ kN$

$L_R = 10^6$ cycles

$a = 3$ for ball bearings

$F_D = F_e = X_i V F_r + Y_i F_a = 0.56 \cdot 1 \cdot 3.57 + 1.31 \cdot 1.3 = 3.7\ kN$

$\frac{F_a}{C_0} = \frac{1.3\ kN}{7.65\ kN} = 0.170 \rightarrow e = 0.34$

$\frac{F_a}{VF_r} = \frac{1.3\ kN}{1 \cdot 3.57\ kN} = 0.364 > e \rightarrow i = 2$

inner ring rotates

$X_2 = 0.56$

$Y_2 = 1.31$

$0.64 \cdot 14.8\ kN \cdot (10^6)^{1/3} = 3.7\ kN \cdot L_D^{1/3}$

$L_D = \left(\frac{0.64 \cdot 14.8 \cdot 100}{3.7}\right)^3 = 1.7 \cdot 10^7$ cycles

PROBLEM No. 5 (18 points)

The close running fit H8/f7 has been selected for a journal bearing with a 30 mm basic size, giving dimensions of $D_{min} = 30$ mm, $D_{max} = 30.033$ mm, $d_{min} = 29.959$ mm and $d_{max} = 29.98$ mm.

SAE 30 oil is used as the lubricant. The shaft rotates at 2000 rpm and the operating temperature is 60 ºC.

The applied load is 3.1 kN.

The bearing length is 30 mm.

Determine the following.

(a) The maximum and minimum clearances

(b) The Sommerfeld number using the average clearance

(c) The coefficient of friction

(d) The power lost to friction

(e) If the bearing was analyzed using the maximum clearance, would the power lost to friction increase or decrease? Briefly justify your answer.

a) $$C_{max} = \frac{D_{max} - d_{min}}{2} = \frac{30.033 - 29.959}{2} = 0.037 \text{ mm}$$

$$C_{min} = \frac{D_{min} - d_{max}}{2} = \frac{30 - 29.98}{2} = 0.01 \text{ mm}$$

$$C_{avg} = \frac{0.037 + 0.01}{2} = 0.0235 \text{ mm}$$

b) $$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{15}{0.0235}\right)^2 \cdot \frac{26 \cdot 10^{-3}\ \text{Pa} \cdot 33.33}{3.44 \cdot 10^{6}\ \text{Pa}} = 0.102$$

$r = 15$ mm

$c = 0.0235$ mm

PROBLEM No. 5 (continued)

$\mu = 26 \text{ mPa}\cdot\text{s}$

$N = 2000 \frac{\text{rev}}{\text{min}} \cdot \frac{1\text{min}}{60\text{s}} = 33.33 \text{ rev/s}$

$P = \frac{W}{2rl} = \frac{3100 \text{ N}}{0.03\text{m} \cdot 0.03\text{m}} = 3.44 \text{ MPa}$

c) Figure 12-17 w/ $S = 0.1$ and $l/d = 1$

$\frac{r}{c} f = 2.8$

$f = 2.8 \cdot \frac{0.0235\text{mm}}{15 \text{ mm}} = 0.0044$

d) power $= fWrN = 0.0044 \cdot 3100\text{N} \cdot 0.015\text{m} \cdot 33.33 \frac{\text{rev}}{\text{s}} \cdot \frac{2\pi \text{ rad}}{\text{rev}}$

$= 42.7 \text{ W}$

e) @ maximum clearance $S = \left(\frac{15}{0.037}\right)^2 \cdot \frac{26 \cdot 10^{-3} \cdot 33.33}{3.44 \cdot 10^6} = 0.04$

$\frac{r}{c} f = 1.6 \rightarrow$ coefficient of friction decreases. and power loss will decrease.