ME 35400 – Machine Design I

Spring 2021: Final Exam

Monday, May 03, 2021

Name: ______________________________ **Email:** ______________________________

Instructions:

- This is an open book, closed notes exam.
- The use of any online sources, such as Chegg.com is not allowed. Failure to adhere to this will result in a score of 0 on this exam.
- Once completed, please upload a scanned PDF file on Gradescope.

Problem 1 (25 points)

A spur-gear set with gear ratio $m_G = 5$, a diametral pitch of $P = 12$ teeth per inch transmits 5 horsepower. The rotational speed of the pinion is 1000 rpm. The center distance between the gear and the pinion is 6 inches. All of the teeth are full-depth with a 20^o pressure angle.

The following parameters and AGMA factors are known for pinion:

- Overload Factor, $K_o = 1.25$
- Rim Thickness Factor, $K_B = 1.0$
- Dynamic Factor, $K_v = 1.1$
- Surface Condition Factor, $C_f = 1.0$
- Size Factor, $K_s = 1.1$
- Geometry Factor, $I = 0.13$
- Load Distribution Factor, $K_m = 1.2$
- Elastic Coefficient, $C_p = 2300\ \sqrt{\text{psi}}$
- Face width, $F = 1$ inch

Calculate

(i) the pitch diameters of the pinion and the gear.
(ii) the number of teeth on pinion and the gear.
(iii) the minimum number of teeth for the pinion to prevent interference. Is the interference constraint met for pinion? Why or why not?
(iv) the AGMA bending stress for the pinion.
(v) the AGMA contact stress for the pinion.
(vi) the probability of failure of the pinion due to bending if it is manufactured using Grade 1 carburized and hardened steel. It is designed to last for 10^7 cycles with a bending factor of safety is 2.0 under room temperature.

Problem #1

Given:

Spur gears

Gear ratio : $m_G = 5$

$P = 12$ teeth / inch

Power $= 5$ hp $= H$

Center distance $=$ 6 inches

Full depth,

20° pressure angle

1000 rpm

$K_o = 1.25$

$K_B = 1.0$

$K_v = 1.1$

$C_f = 1.0$

$K_s = 1.1$

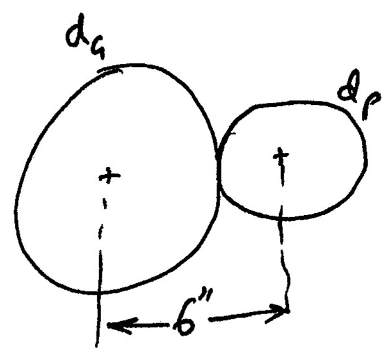
$I = 0.13$

$K_m = 1.2$

$C_p = 2300\sqrt{psi}$

$F =$ 1 inch

(i) $$\frac{d_G + d_P}{2} = 6 \quad \text{— (1)}$$



and $$\frac{d_G}{d_P} = 5 \quad \text{— (2)}$$ (Gear ratio)

$\Rightarrow d_G = 5 d_P$

and $d_G + d_P = 12$

$\Rightarrow 5 d_P + d_P = 12$

$\boxed{d_P = 2"}$ diameter of the pinion

$\boxed{d_G = 10"}$ diameter of the Gear

(ii) $N_G = P d_G$

$= 12(10)$

$\boxed{N_G = 120}$ number of teeth in the gear

$N_p = P d_p$

$= 12(2)$

$\boxed{N_p = 24}$ number of teeth in the pinion.

(iii) The minimum number of teeth in the pinion to prevent interference is:

$$N_p = \frac{2K}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right) \quad K = 1 \text{ for full depth}$$

$$= \frac{(2)(1)}{(1+2\times5)\sin^2 20^\circ}\left[5 + \sqrt{5^2 + (1+10)\sin^2\phi}\right]$$

$$= 15.74 \simeq 16$$

Since the number of teeth in the pinion is 24, (>16) it satisfies the interference constraint.

(iv) AGMA Bending stress:

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

eqn(14-15)

where $W^t = \dfrac{33000 H}{V}$

and $V = \dfrac{\pi d n}{12} = \dfrac{\pi (2)(1000)}{12} = 523.6$ ft/min

$\therefore W^t = \dfrac{33000 \times 5}{523.6} = 315.13$ lbf

$$\therefore \sigma = (315.13)(1.25)(1.1)(1.1)\frac{(12)}{1} \cdot \frac{(1.2)(1.0)}{J}$$

$J \leftarrow 0.37$

From Fig 14-6, $J = 0.37$

$$\therefore \boxed{\sigma = 18.55 \text{ kpsi}}$$

(v)
$$\sigma_c = C_p \sqrt{W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I}}$$

$$= 2300 \sqrt{(315.13)(1.25)(1.1)\frac{(1.1)\boxed{1.2}}{(2)(1)} \frac{(1.0)}{(0.134)}}$$

$$I = \frac{\cos\phi_t \sin\phi_t}{2 m_N} \cdot \frac{m_G}{m_G + 1}$$ (eqn 14-23)

m_N = 1 for spur gears

$$\therefore \quad I = \frac{\cos(20)\,\sin(20)}{2\,(1)} \cdot \frac{5}{(5+1)}$$

$$I = 0.134$$

$$\therefore \quad \boxed{\sigma_c = 106.26 \text{ kpsi}}$$

(vi) Grade 1 carburized and hardened steel

$$S_t = 55000 \text{ psi} \qquad (\text{Table } 14\text{-}3)$$

$$S_F = \frac{S_t\, Y_N}{\sigma\,(K_T\, K_R)}$$

$K_T = 1$ (room temp)

$K_R = ?$

$\sigma = 18.55$ kpsi

$Y_N = 1$ for 10^7 cycles.

$S_F = 2.0$ (given)

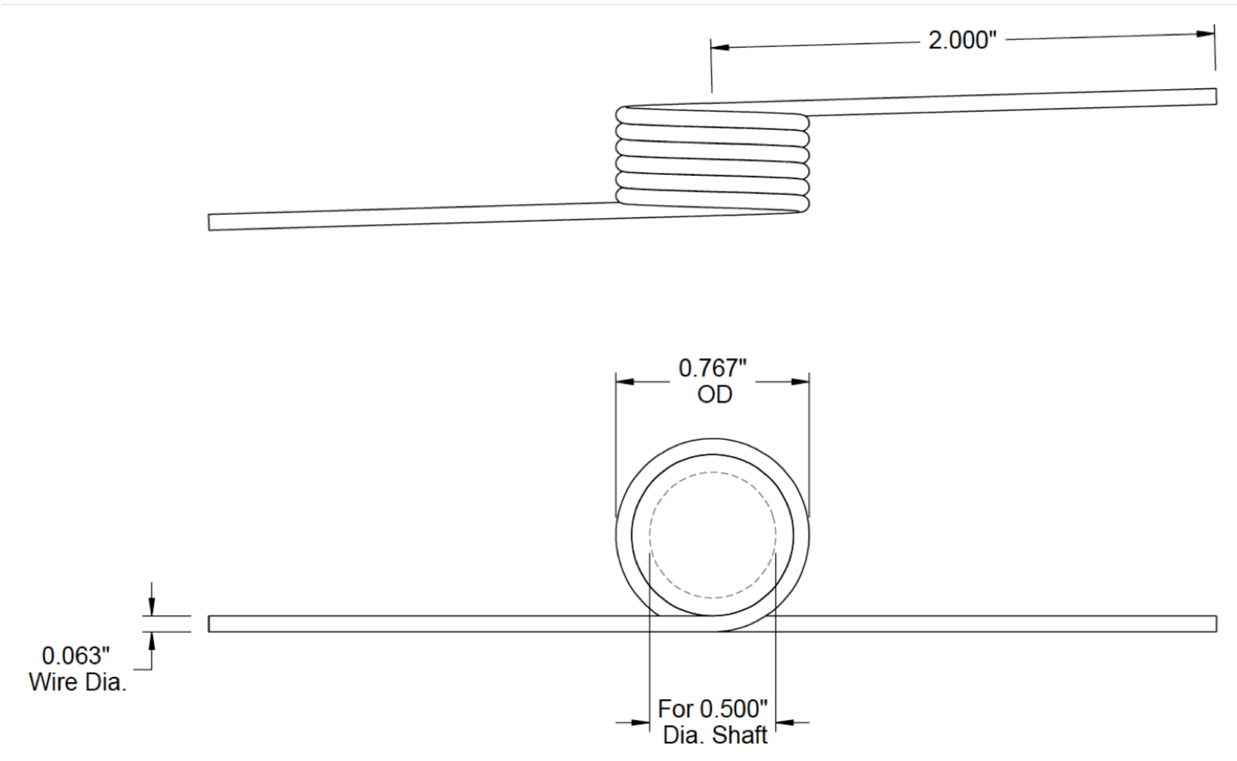
$$\therefore \quad K_R = \frac{S_t\, Y_N}{\sigma\, K_T\, S_F} = \frac{(55000)\cdot(1)}{(18550)\,(1)\,(2)}$$

$$K_R = 1.48$$

from Table 14-10, Reliability is 0.9999 ⇒ Prob of fail

Problem 2 (30 points):

The torsion spring shown in the schematic below is made of music wire.



Calculate:

(i) The angle β and number of body turns (N_b).
(ii) The wire's yield strength (S_y) and ultimate tensile strength (S_{ut}).
(iii) The spring index (C).
(iv) The spring rate (k'). Express your answer in units of torque/turn.
(v) For a maximum load of 5.518 in-lbf, what is the factor of safety guarding against yielding?
(vi) For a factor of safety for infinite life of 1.5 using the Goodman failure criterion, what is the spring's endurance limit (S_e) for a load that varies between 2 and 4 in-lbf?

i) $\beta = 0$ +2 $N_b = 6$ +2 by inspection

ii) $S_y' = 0.78\, S_{ut}$ (Eqn. 10-57)

$$S_{ut} = \frac{A}{d^m} = \frac{201 \text{ kpsi} \cdot \text{in}^{0.145}}{(0.063 \text{ in})^{0.145}} = 300 \text{ kpsi}$$ +2

$$S_y' = 0.78 \cdot 300 \text{ kpsi} = 234 \text{ kpsi}$$ +2

+2 for correct D

iii) $$C = \frac{D}{d} = \frac{OD - d}{d} = \frac{0.767 \text{ in} - 0.063 \text{ in}}{0.063 \text{ in}} = 11.2$$ +1 for correct calculation of C

iv) $$k' = \frac{d^4 E}{10.8 D N_a}$$

$d = 0.063$ in

$E = 29$ Mpsi (Table 10-5) +1

$D = OD - d = 0.704$ in

+1 for $l_1 = l_2 = 2$ in

$$N_a = N_b + \frac{l_1 + l_2}{3\pi D} = 6 + \frac{2 \text{ in} + 2 \text{ in}}{3\pi \cdot 0.704 \text{ in}} = 6.6 \text{ coils}$$ +1

$$k' = \frac{(0.063 \text{ in})^4 \cdot 29 \cdot 10^6 \text{ lbf/in}^2}{10.8 \cdot 0.704 \text{ in} \cdot 6.6} = 9.1 \text{ lbf-in/turn}$$ +2

v) $$n = \frac{S_y'}{\sigma} \qquad n = 1 \rightarrow \sigma = S_y' = 234 \text{ kpsi}$$ +2

$$\sigma = K_i \cdot \frac{32 Fr}{\pi d^3} = 1.07 \cdot \frac{32 \cdot Fr}{\pi \cdot (0.063 \text{ in})^3} = 234 \text{ kpsi} \rightarrow Fr = 5.4 \text{ in-lbf}$$ +2

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = 1.07 \quad +2$$

vi) $$\sigma_{min} = K_i \cdot \frac{32Fr}{\pi d^3} = 1.07 \cdot \frac{32 \cdot 2 \text{ in.lbf}}{\pi d^3} = 87.3 \text{ kpsi} \quad +2$$

$$\sigma_{max} = K_i \cdot \frac{32Fr}{\pi d^3} = 1.07 \cdot \frac{32 \cdot 4 \text{ in.lbf}}{\pi d^3} = 174.6 \text{ kpsi} \quad +2$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 43.6 \text{ kpsi} \quad +1$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 130.9 \text{ kpsi} \quad +1$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}$$

$$\frac{1}{1.5} = \frac{43.6}{S_e} + \frac{130.9}{300} \rightarrow S_e = 189.5 \text{ kpsi} \quad +2$$

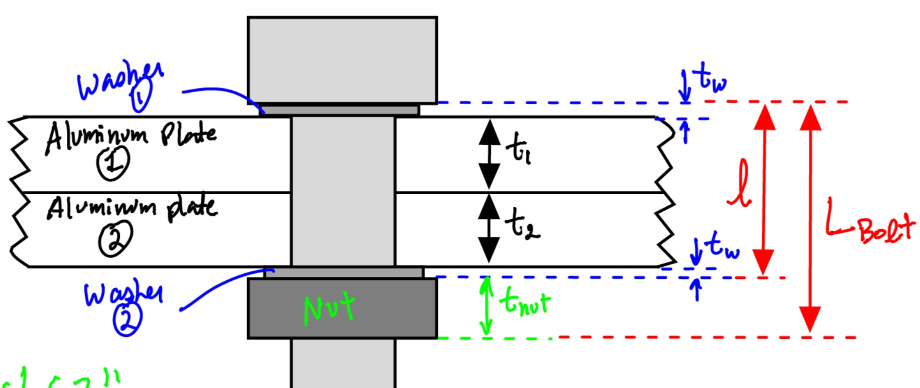
Problem 3 (30 points)

Given: Two identical aluminum plates are compressed with one bolt and nut. Washers are used under the head of the bolt and with the nut, as shown in the figure. The system has the following properties:

- Bolt: steel; UNC-$\frac{1}{4}$-20; SAE Grade 7; rolled threads; S_{ut} = 120 kpsi; E_{bolt} = 30 Mpsi
- Washers: steel; ID = 0.281 in; OD = 0.625 in; thickness = 0.065 in; E_{washer} = 30 Mpsi
- Nut: steel; height = $\frac{7}{32}$ in; E_{nut} = 30 Mpsi
- Plates: aluminum; E_{plate} = 10.3 Mpsi; S_{ut} = 47 kpsi; S_y = 25 kpsi; thickness = 2.25 in
- Loading: Preload = 3000 lbs; External loading fluctuates from 1000 lbs to 2000 lbs

Find: Determine the following:

(a) A suitable length for the bolt, rounded up to the nearest $\frac{1}{4}$ in.
(b) The overall stiffness of the members, k_m.
(c) The joint stiffness, C.
(d) If the washers are removed from the joint, state effect that will that have on the overall joint stiffness C (i.e., will it increase, decrease, or be unchanged?). Justify your answer.
(e) For the bolt, determine the following:
  (i) The load factor guarding against joint separation, η_o
  (ii) The load factor guarding against exceeding the proof load, η_L
  (iii) The yielding factor of safety, η_p
  (iv) The factor of safety guarding against fatigue using the modified Goodman criterion, η_f



(a) $L_{Bolt} \geq \overbrace{2t_w + t_1 + t_2}^{l = 4.63''} + t_{Nut}$

(2 pts) $\geq 2(0.065'') + (2.25'') + (2.25'') + (7/32'')$

≥ 4.8487 $\therefore$ rounding to nearest $1/4''$ $\Rightarrow$ $\underline{L_{Bolt} = 5''}$

(b) Stiffness of the members includes the two steel washers and two aluminum plates. Since washers and plates are the same, just need to calculate each once

Member Stiffness: $K_m = \left[\frac{1}{K_w} + \frac{1}{K_w} + \frac{1}{K_{plate}} + \frac{1}{K_{plate}} \right]^{-1}$

eq 8-20 (for $\alpha = 30°$)

$$K = \frac{0.5774 \pi E d}{\ln \frac{(1.155t + D - d)(D + d)}{(1.155t + D - d)(D - d)}}$$

For Washer : $E = 30$ Mpsi
$t = t_w = 0.065"$
$d = ID = 0.281"$
$D = 1.5 d_{Bolt} = 1.5(0.25") = 0.375"$

OK If choose $d = d_{bolt} = 1/4"$ then get $K_w = 38 \, M \frac{lbf}{in}$

(3 pts) $\therefore$ $K_w = 31.95 \times 10^6 \frac{lbf}{in} = 31.95 \, M \frac{lbf}{in}$

For plate: $E = 10.3$ Mpsi
$t = t_1 = t_2 = 2.25"$
$d = 0.25"$
$D = 1.5d + 2 t_w \tan\alpha$
$= 1.5(0.25") + 2(0.065") \tan(30°) = 0.4501"$

(3 pts) $\therefore$ $K_{plate} = 4.29 \, M \, lbf/in$

$$K_m = \left[\frac{1}{31.95} + \frac{1}{31.95} + \frac{1}{4.29} + \frac{1}{4.29} \right]^{-1}$$

(2 pts) $\underline{K_m = 1.89 \, M \, lbf/in}$ $\left(K_m = 1.92 \, M \frac{lbf}{in} \right)$

(c) (5 pts) $C = \dfrac{k_b}{k_b + k_m} \Rightarrow$ Need k_b: eq 8-17 $\quad k_b = \dfrac{A_d A_t E_{bolt}}{A_d l_t + A_t l_d}$

From Table 8-7

$L_T = 2d + 1/4'' \quad (L \le 6'')$

$= 2(0.25'') + 1/4'' = 0.75''$

$l_d = L - L_T = 5'' - 0.75''$

$\therefore l_d = 4.25''$

$l_t = l - l_d = 4.63'' - 4.25''$

$= 0.38''$

$A_d = \dfrac{\pi d^2}{4} = \dfrac{\pi (0.25'')}{4} = 0.0491 \text{ in}^2$

$A_t = 0.0318 \text{ in}^2$ (Table 8-2 for UNC 1/4-20)

$E_{bolt} = 30$ Mpsi (given)

$$\therefore k_b = \frac{(0.0491 \text{ in}^2)(0.0318 \text{ in}^2)(30 \times 10^6 \text{ lbf/in}^2)}{(0.0491 \text{ in}^2)(0.38'') + (0.0318 \text{ in}^2)(4.25'')}$$

$$k_b = 0.305 \text{ M } \frac{\text{lbf}}{\text{in}}$$

$$C = \frac{0.305 \text{ M lbf/in}}{(0.305 + 1.89) \text{ M lbf/in}} = \underline{0.1386}$$

(0.136)

(d) If remove washers, joint stiffness should decrease

(2 pts) Proof/Check: w/o washers $\Rightarrow$ $k_m = \left[\dfrac{1}{4.29} + \dfrac{1}{4.29}\right]^{-1} = 2.14 \text{ M } \dfrac{\text{lbf}}{\text{in}}$

$$\therefore C = \frac{0.305}{(0.305 + 2.14)} = 0.1242$$

joint stiffness <u>decreases</u> ✓

Technically need to update k_b for new shorter bolt length and corresponding l_t and l_d but just above will suffice [$L = 4.75''$, $l_d = 4''$, $l_t = 0.5'' \Rightarrow k_b = 0.308 \Rightarrow C = 0.126$]

(e) $F_i = 3000$ lbs (given)
$C = 0.1386$
$A_t = 0.0318 \text{ in}^2$ } from part (c)

From Table 8-9 (Grade 7) $S_p = 105$ kpsi

$P = P_{max} = 2000$ lbs

(3 pts) (i) eq 8-30

$$n_o = \frac{F_i}{P(1-C)} = \frac{3000 \text{ lbf}}{2000 \text{ lbf}(1-0.1386)} = \underline{1.74}$$

(3 pts) (ii) eq 8-29

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{(105\times10^3 \text{ lbf/in}^2)(0.0318 \text{ in}^2) - 3000 \text{ lbf}}{(0.1386)(2000 \text{ lbf})} = \underline{1.22}$$

(3 pts) (iii) eq 8-28

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{(105\times10^3 \frac{\text{lbf}}{\text{in}^2})(0.0318 \text{ in}^2)}{(0.1386)(2000 \text{ lbs}) + 3000 \text{ lbf}} = \underline{1.02}$$

(4 pts) (iv) eq 8-38

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)}$$

$$\sigma_i = \frac{F_i}{A_t} = 94.3 \text{ kpsi}$$

eq 8-36 $$\sigma_m = \frac{C(P_{max} + P_{min})}{2A_t} + \frac{F_i}{A_t} = 100 \text{ kpsi}$$

$P_{max} = 2000$ lbf
$P_{min} = 1000$ lbf
$S_{ut} = 120$ kpsi
} given

eq 8-35 $$\sigma_a = \frac{C(P_{max} - P_{min})}{2A_t} = 218 \text{ kpsi}$$

Table 8-17 SAE grade 7 rolled threads
$S_e = 20.6$ kpsi

$$\therefore n_f = \frac{(20.6)(120-94.3)}{(120)(218)+(20.6)(100-94.3)} = \underline{1.33}$$

Problem 4 (15 points)

A multi-disc plate clutch has six friction pairs (N = 6) with a disk outer diameter of 7.0 inches and an inner diameter of 4.0 inches. The coefficient of friction is 0.25. The maximum pressure is 120 psi.

(i) Calculate the axial force F and the transmitted torque, T using the uniform wear model.
(ii) Calculate the axial force F and the transmitted torque, T using the uniform pressure model.
(iii) What is the torque capacity of this clutch when it is new? Explain why.

Problem # 4

$N = 6$

$D = 7.0$ inches

$d = 4.0$ inches

$f = 0.25$

$P_a = 120$ psi

(i) Unif. wear:

$$F = \frac{\pi P_a}{2} d(D-d) = \frac{\pi(120)(4)}{2}(7-4) = 2.262\times10^3 \text{ lbf}$$

$$T = N\frac{Ff}{4}(D+d) = \frac{(6)(2.262\times10^3)(0.25)}{4}(7+4)$$

$$= 9.33\times10^3 \text{ in-lb}$$

(ii) Unif. pressure:

$$F = \frac{\pi P_a}{4}(D^2-d^2) = 3.11\times10^3 \text{ lbf}$$

$$T = N\frac{Ff}{3}\,\frac{D^3-d^3}{D^2-d^2} = 6\frac{(3.11\times10^3)(0.25)}{3}\left(\frac{7^3-4^3}{7^2-4^2}\right)$$

$$= 13.15\times10^3 \text{ in-lb}$$

(iii) The torque capacity for a new clutch is 13.15×10^3 in-lb. because unif. pressure is more accurate.