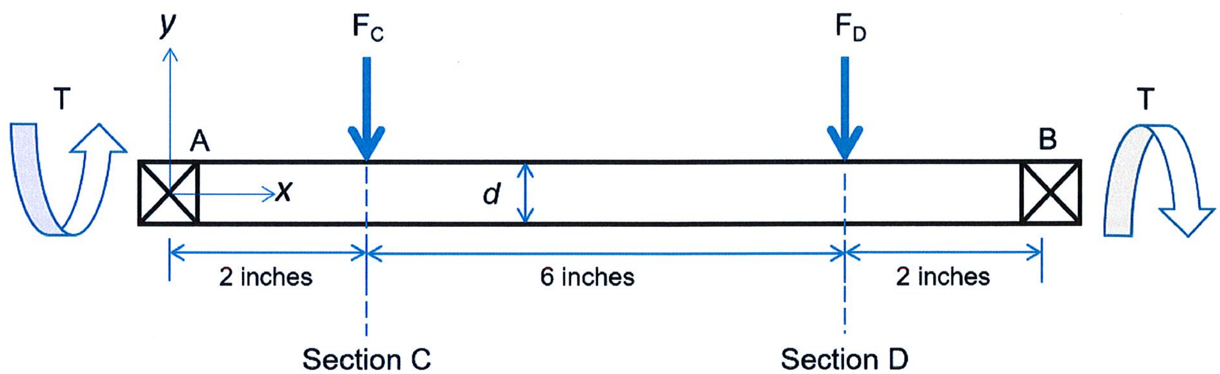
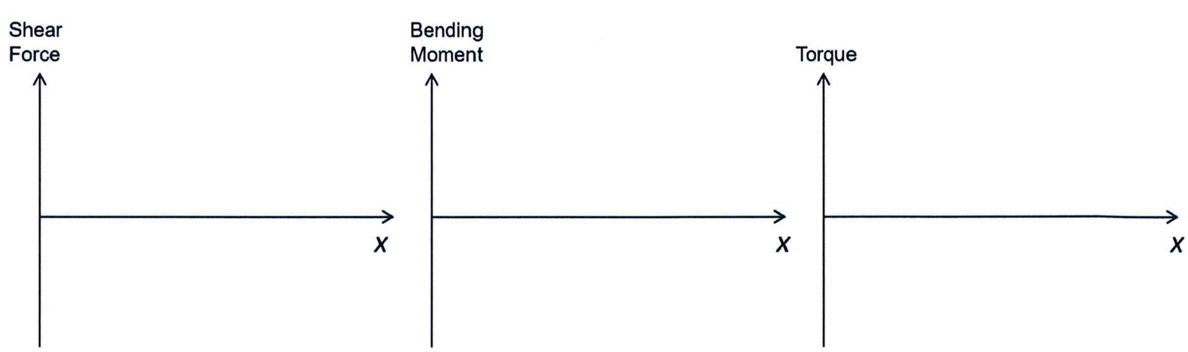
Problem 1 (30 points):

The steel shaft shown below is rotating at 1000 rpm, and is simply-supported at bearings A and B. The shaft has a circular cross-section with an unknown diameter, d. The torque, T, transmitted by the shaft varies from $T_{\min} = -2000$ in-lb and $T_{\max} = +6000$ in-lb. Two constant transverse loads with magnitudes $F_C = 3000$ lb and $F_D = 3000$ lb and directions shown in the figure act on the shaft at sections C and D respectively.

Assume $S_{ut} = 140$ kpsi and $S_y = 120$ kpsi, $S_e = 60$ kpsi (fully corrected). Stress concentration effects at section D are due to features in the shaft for mounting a gear. Assume that the fatigue stress concentration factors for the critical point at section D are $K_f = 1.60, K_{fs} = 1.40$.

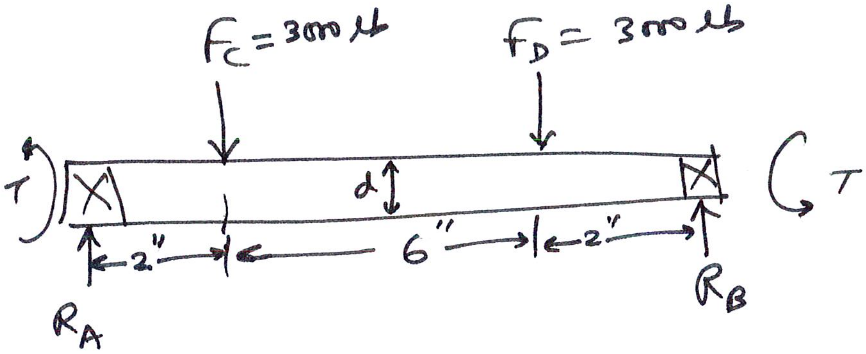
(i) Calculate the reaction forces at the bearings A and B.
(ii) Sketch the shear force, bending moment, and torque diagrams for the shaft.
(iii) Calculate the diameter, d, of the shaft for an infinite-life fatigue factor of safety of 2.0 at section D, using the Goodman failure criterion.
(iv) Calculate the diameter, d, of the shaft for a static factor of safety of 2.0 at section D. Include stress concentration for this material.
(v) Which failure criteria (static or fatigue) governs the design at section D, and why?





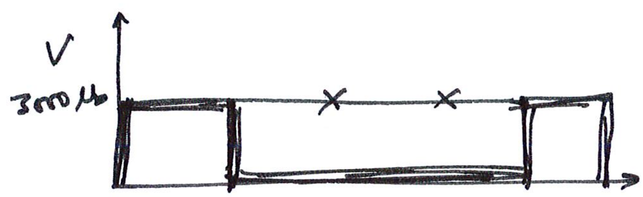
Solution

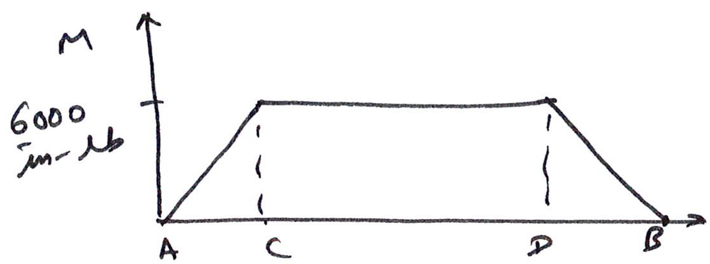
Problem #1



(i) Due to symmetry, $R_A = R_B = 3000$ lb

(ii)





$M = (3000)(2")$

$M = 6000$ in-lb

(iii) The bending moment is completely reversed.

$\therefore \quad M_m = 0$

$M_a = 6000$ in-lb

$T_{min} = -2000$ in-lb

$T_{max} = +6000$ in-lb

$$\Rightarrow \begin{cases} T_m = \dfrac{6000 - 2000}{2} = 2000 \text{ in-lb} \\ T_a = \dfrac{6000 + 2000}{2} = 4000 \text{ in-lb} \end{cases}$$

The diameter using the modified Goodman line is:

$$d = \left[\frac{16 n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]^{1/3}$$ eqn (7-8)

where $A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$

$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$

$\Rightarrow \quad A = \sqrt{4[(1.6)(6000)]^2 + 3[(1.4)(4000)]^2} = 2151.1$ psi

$B = \sqrt{4(0) + 3[(1.4)(2000)]^2} = (\sqrt{3})(1.4)(2000)$

$= 4849.7$ psi

$$\therefore \quad d = \left[\frac{16 \times 2}{\pi}\left(\frac{2.1511}{60} + \frac{4.8497}{140}\right)\right]^{1/3}$$

$\Rightarrow \quad \underline{d = 0.8764''}$

(iv) Static factor of Safety:

$$n_y = \frac{S_y}{\sigma'_{max}}$$ (eqn 7-16)

where $$\sigma'_{max} = \left[\left(\frac{32 K_f M_{max}}{\pi d^3}\right) + 3\left(\frac{16 K_{fs} T_{max}}{\pi d^3}\right)\right]^{1/2}$$

eqn (7-15)

$$\Rightarrow \quad \sigma'_{max} = \left[\left(\frac{32 \times 1.6 \times 6000}{\pi d^3} \right)^2 + 3 \left(\frac{16 \times 1.4 \times 6000}{\pi d^3} \right)^2 \right]^{1/2}$$

$$\sigma'_{max} = \frac{1}{d^3} \left[1.2269 \times 10^5 \right]$$

$$\Rightarrow \quad 2.0 = \frac{120{,}000 \, (d)^3}{1.2269 \times 10^5}$$

$$\Rightarrow \quad d^3 = \frac{2.0 \times 1.2269 \times 10^5}{120{,}000}$$

$$\Rightarrow \quad \boxed{d = 1.2693}$$

(V) The static failure governs the design because the diameter is greater.

Problem 2 (25 points):

A 1.5 inch diameter shaft transmits a time-varying torque T through a standard rectangular key with a width of $\frac{3}{8}$ inch and a height of $\frac{1}{4}$ inch. The depth of the key-seat in the shaft is $\frac{1}{8}$ inch. The torque T fluctuates between $T_{\text{min}} = +2000$ in-lb and $T_{\text{max}} = +4000$ in-lb.

The material properties of the key are as follows: $S_{ut} = 60$ kpsi, $S_y = 50$ kpsi, and $S_e = 30$ kpsi. The endurance limit S_e is fully corrected and was calculated with $k_c = 1$.

(i) Draw the free body diagram of the key, and show the planes of shear and crushing failure.

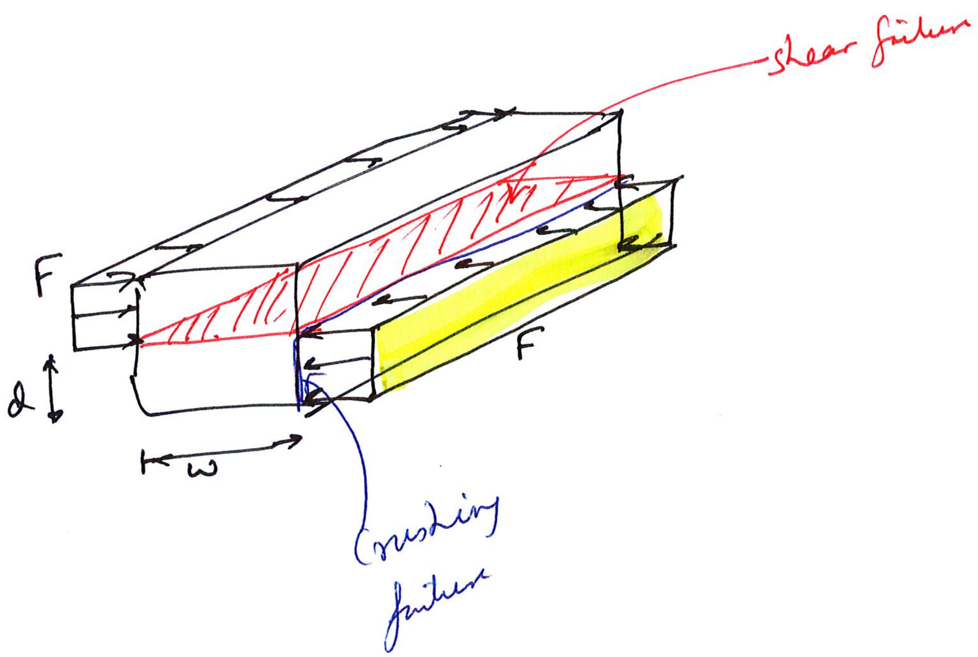
(ii) Calculate the length of the key required to achieve a *static* factor of safety of 2.0 under *shear failure.*

(iii) Calculate the length of the key required to achieve a *static* factor of safety of 2.0 under *crushing failure.*

(iv) Calculate the length of the key required to achieve a *fatigue* factor of safety of 2.0 under *shear failure* using the Goodman criterion.

(v) Does the key have an optimal aspect ratio? If not, calculate the optimal width of the key, assuming the same height ($= \frac{1}{4}$ inch).

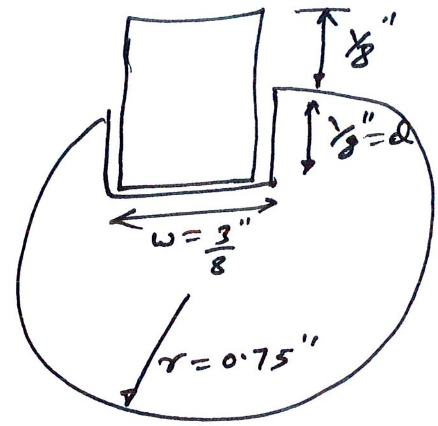
(i)



Problem #2:

shaft dia $= 1.5''$

$\therefore \quad r = 0.75''$



$$\text{key} \begin{cases} w = \frac{3}{8} \text{ inch} \\ d = \frac{1}{8} \text{ inch} \end{cases}$$

$$\begin{cases} T_{min} = +2000 \text{ in-lb} \\ T_{max} = +4000 \text{ in-lb} \end{cases}$$

$$\Rightarrow \quad T_a = \frac{4000 - 2000}{2} = 1000 \text{ in-lb}$$

$$T_m = \frac{4000 + 2000}{2} = 3000 \text{ in-lb.}$$

(ii) Static (yield) FOS under shear failure:

$$n_{y,shear} = \frac{S_y}{\sigma'_{max}} = \frac{S_y \, (r l w)}{\sqrt{3}\,(T_m + T_a)}$$

$$(n_y)_{shear} = \frac{(50{,}000)\,(0.75)\,(l)\,\frac{(3)}{(8)}}{\sqrt{3}\,(4000)} = 2$$

$$\Rightarrow \quad l = 0.9853 \text{ inch}$$

(iii) Static (Yield) fos under crushing:

$$(n_y)_{crush} = \frac{S_y\,(r\,l\,d)}{(T_m + T_a)} = 2.0$$

$$\Rightarrow \quad l = \frac{(2.0)\,(T_m + T_a)}{S_y\,(r\,d)} = \frac{(2.0)(4000)(8)}{(50{,}000)(0.75)(1)}$$

$$\boxed{l = 1.7067 \text{ inches}}$$

(iv) Fatigue FOS under shear failure:

$$\frac{1}{n_{f,\,shear}} = \frac{\sqrt{3}}{r\,l\,w}\left(\frac{T_m}{S_{ut}} + \frac{T_a}{S_e}\right)$$

$$\Rightarrow \quad l = \frac{\sqrt{3}\,(n_{f,\,shear})}{r\,w}\left(\frac{T_m}{S_{ut}} + \frac{T_a}{S_e}\right)$$

$$= \frac{(\sqrt{3})(2)(8)}{(0.75)(3)}\left(\frac{3000}{60{,}000} + \frac{1000}{30{,}000}\right)$$

$$\boxed{l = 1.0264 \text{ inches}}$$

(v) No, the optimal aspect ratio is:

$$d = 0.577w$$

$$\Rightarrow \quad w = \frac{d}{0.577} = \frac{1}{(8)(0.577)} = \underline{0.2166 \text{ inch}}$$

$\therefore$ Optimal width = $\underline{0.2166 \text{ inch}}$

ALTERNATE SOLUTION for part (iv):

(iv) $$\frac{1}{n_{f,shear}} = \frac{1}{r l w}\left(\frac{T_m}{0.67 S_{ut}} + \frac{T_a}{0.59 S_e}\right)$$

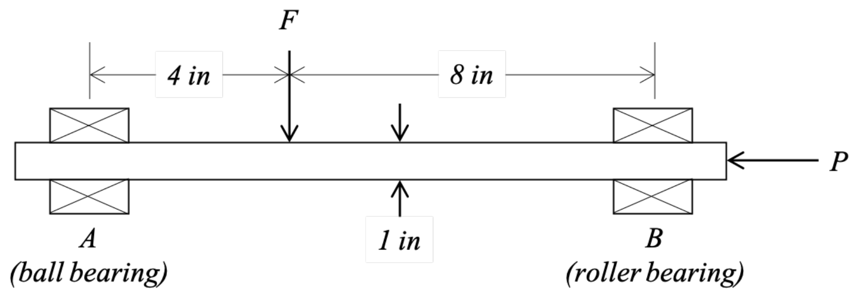
$$\Rightarrow \quad l = \frac{n_{f,shear}}{r w}\left(\frac{T_m}{0.67 S_{ut}} + \frac{T_a}{0.59 S_e}\right)$$

$$= \frac{(2)(8)}{(0.75)(3)}\left(\frac{3000}{0.67(60{,}000)} + \frac{1000}{(0.59)(30{,}000)}\right)$$

$$\boxed{l = 0.9324}$$

Problem 3 (~~25~~ 20 points):

The rotating shaft shown below is supported by a ball bearing at A and by a roller bearing at B. The ball bearing at A supports both a radial load and an axial load, where the axial load is $P = 400$ lbf. The roller bearing at B supports a radial load only. The bearings' inner rings rotate.



(i) For $F = 1800$ lbf, determine the radial force supported by the ball bearing at A and the radial force supported by the roller bearing at B.

(ii) Catalog data for a deep groove ball bearing that will be used for bearing A are as follow, where the catalog rating life is 10^6 cycles. Determine the bearing life in revolutions for 90% reliability.

Trade No.	For Shaft Dia.	For Housing ID	Wd.	Ring Material	Radial Load Cap., lbs. Dynamic	Static
R16	1"	2"	3/8"	Steel	2,400	1,450
					C_{10}	C_0

(iii) The roller bearing at B is to have the same life as the the ball bearing at A. Determine the basic dynamic load rating for the roller bearing at B, for 90% reliability where the catalog rating life is 90×10^6 revolutions.

+1

i) $\Sigma M_A = 0 \rightarrow -1800 \cdot 4 + 12 \cdot F_B = 0 \rightarrow F_B = 600$ lbf

$\Sigma F_y = 0 \rightarrow F_A = 1200$ lbf +1

ii) for 90% reliability

$F_R L_R^{1/a} = F_D L_D^{1/a}$ Eqn 11-2a

$F_R = 2400$ lbf (from catalog) +2

$L_R = 10^6$ (given) +1

$a = 3$ for ball bearing +1

$F_D = F_e = X_i V F_r + Y_i F_a = 1 \cdot 1 \cdot 1200 + 0 \cdot 400 = 1200$ lbf +1 for equivalent load

$$\frac{F_a}{C_0} = \frac{400 \text{ lbf}}{1450 \text{ lbf}} = 0.2758 \approx 0.28 \rightarrow e = 0.38 \text{ from Table 11-1}$$

+1 for F_a
+2 for C_0
+2 for choice of e

$$\frac{F_a}{VF_r} = \frac{400 \text{ lbf}}{1 \cdot 1200 \text{ lbf}} = 0.33 < e \rightarrow i = 1, \; X_1 = 1, \; Y_1 = 0$$

+2 for $\frac{F_a}{VF_r} < e$ and choice of $i = 1$

$L_D = 8 \cdot 10^6$ revolutions.

(ii) $F_R L_R^{1/a} = F_D L_D^{1/a}$

$F_R = ?$

$L_R = 90 \cdot 10^6$ rev. +1

$a = 10/3$ +1

$F_D = 600$ lbf +1

$L_D = 8 \cdot 10^6$ rev. +1

$$F_R = \frac{600 \cdot (8 \cdot 10^6)^{1/10/3}}{(90 \cdot 10^6)^{1/10/3}} = 290.3 \text{ lbf} \quad +2$$

Problem 4 (25 points):

Given: A full journal bearing is 2.5 inches long with a l/d ratio of 0.5. The bushing bore has a diameter of 5.005 inches. The load is 525 lbf and the journal speed is 800 rev/min. The operating temperature is 145°F and SAE 40 lubricating oil is used.

Find: Determine the following:

(a) The Sommerfeld number
(b) The minimum film thickness, h_o, and the eccentricity of the film, e
(c) The coefficient of friction, f
(d) The lubricant side flow rate, Q_s
(e) The power loss due to friction in units of horsepower (hp)
(f) The operating temperature is now 180°F. Choose the appropriate SAE grade oil to use in order to keep the power loss due to friction the same as determined above. Justify your answer.

Given: $l = 2.5''$ $\frac{l}{d} = 0.5$ $d_{Bore} = 5.005''$ $W = 525\ lbf$ $N = 800\ \frac{rev}{min}$

SAE 40 @ 145°F

Solution Need to calculate S first in order to use charts to determine other parameters

(a) For SAE 40 @ 145°F

Fig 12-2: $\mu = 5\ \mu reyn$ (1 pt)

$N = 800\frac{rev}{min} * \frac{1 min}{60 sec} = 13.33\frac{rev}{s}$ (1 pt)

since $\frac{l}{d} = 0.5 \Rightarrow d = \frac{l}{0.5} = \frac{2.5''}{0.5} = 5'' = d_{max\ journal}$

$c = \frac{d_{Bore} - d_{max}}{2} = \frac{5.005'' - 5''}{2} = 0.0025''$ (1 pt)

$r = \frac{d}{2} = 5/2 = 2.5''$ $\frac{r}{c} = \frac{2.5''}{0.0025''} = 1000$

eq 12-7 $P = \frac{W}{2rl} = \frac{525\ lbf}{(2*2.5'')(2.5'')} = 42\ psi$ (1 pt)

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

(1 pt)

$$= \frac{(1000)^2 (5\times10^{-6})(13.33)}{42}$$

$S = 1.59 \approx 1.6$ (1 pt)

(a) 6 pts

(b) Fig 12-15 for $S = 1.6$ and $l/d = 0.5$ (b) 4 pts

$\frac{h_0}{c} = 0.74$ (1 pt) $\varepsilon = 0.26 = \frac{e}{c}$ (1 pt)

$\therefore h_0 = 0.74(c)$
$= 0.74(0.0025'')$

$\underline{h_0 = 0.00185''}$ ✓ (1 pt)

$\therefore e = \varepsilon c = (0.26)(0.0025'')$

$\underline{e = 0.00065''}$ ✓ (1 pt)

(c) Figure 12-17 (3 pts)

$\frac{r}{c} f = 3.2 \times 10 = 32$ (2 pts)

$\therefore f = 32/(r/c) = \frac{32}{1000} = 0.032$ (1 pt.)

(d) Fig 12-18 (5 pts)

$\frac{Q}{rcNl} = 3.8$ (1 pt)

$\therefore Q = 3.8\, rcNl$

$= 3.8(2.5'')(0.0025'')(13.33\,\frac{rev}{s})(2.5'')$ (1 pt.)

$Q = 0.791\ in^3/s$ (1 pt.)

Fig 12-19

$\frac{Q_s}{Q} = 0.38$ (1 pt)

$\therefore Q_s = 0.38\,Q$

$= 0.38(0.791\,\frac{in^3}{s})$

$\underline{Q_s = 0.300\,\frac{in^3}{s}}$ ✓ (1 pt)

(e) Power loss due to friction (4 pts)

Frictional torque, $T = fWr = (0.032)(525\ lbf)(2.5'')$

$T = 42\ lbf\cdot in$ (1 pt.)

In rad/s

$P_{loss} = TN = (42\ lbf\cdot in)(13.33\,\frac{rev}{s})\left(\frac{2\pi\,rad}{1\,rev}\right) = 3517.7\,\frac{lbf\cdot in}{s}$ (1 pt.)

$1\ hp = 550\,\frac{ft\cdot lbf}{s}$

$\therefore 3517.7\,\frac{lbf\cdot in}{s} \times \frac{1\,ft}{12\,in} \times \frac{1\,hp}{550\,\frac{ft\cdot lbf}{s}} = \underline{0.533\ \ hp}$ ✓ (2 pts)

(f) The operating temperature defines the oil viscosity (μ) (Fig 12-2)

From above, to keep the power loss the same, need the same f. (3 pts) To get same f, need the same S. To get the same S, we need the same $\mu = 5\ \mu reyn$ for New operating temperature of 180°F. (2 pts)

(1 pt) $\therefore$ choose SAE 60 ✓ (Fig 12-2)