EQUATIONS

Axial stress

$$\sigma = \frac{F}{A}$$

Normal stress for beam in bending

$$\sigma = \frac{Mc}{I}$$

$$I = \frac{bh^3}{12} \text{ (rectangular cross-section)}$$

$$I = \frac{\pi r^4}{4} = \frac{\pi}{64}d^4 \text{ (circular cross-section)}$$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{64}(d_o^4 - d_i^4) \text{ (hollow round cross-section)}$$

Maximum transverse shear stress

$$\tau_{max} = \frac{3V}{2A} \text{ (rectangular cross-section)}$$

$$\tau_{max} = \frac{4V}{3A} \text{ (circular cross-section)}$$

$$\tau_{max} = \frac{2V}{A} \text{ (hollow, thin-walled round cross-section)}$$

$$\tau_{max} \approx \frac{V}{A_{web}} \text{ (thin-walled I-beam)}$$

Torsion

$$\tau = \frac{T\rho}{J}$$

$$\tau_{max} = \frac{Tr}{J}$$

$$J = \frac{\pi r^4}{2} = \frac{\pi}{32}d^4 \text{ (circular cross-section)}$$

$$J = \frac{\pi}{2}(r_o^4 - r_i^4) = \frac{\pi}{32}(d_o^4 - d_i^4) \text{ (hollow round cross-section)}$$

EQUATIONS (continued)

Ductile Coulomb-Mohr (DCM) theory

$$\frac{1}{n} = \frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}}$$

Maximum shear stress (MSS) theory for ductile materials

$$n = \frac{S_y}{2\tau_{max}} = \frac{S_y}{\sigma_3 - \sigma_1}$$

Distortion energy (DE) theory for ductile materials

$$n = \frac{S_y}{\sigma'}$$

$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}}$$

$$\sigma' = \frac{1}{\sqrt{2}} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)}$$

Brittle Coulomb-Mohr (BCM) theory

$$\frac{1}{n} = \frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}}$$

Modified Mohr (MM) theory for brittle materials (plane stress)

$$n = \frac{S_{ut}}{\sigma_A} \qquad \text{for } \sigma_A \geq \sigma_B \geq 0 \text{ and for } \sigma_A \geq 0 \geq \sigma_B \text{ where } \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$$

$$\frac{1}{n} = \frac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} \qquad \text{for } \sigma_A \geq 0 \geq \sigma_B \text{ where } \left|\frac{\sigma_B}{\sigma_A}\right| > 1$$

$$n = -\frac{S_{uc}}{\sigma_B} \qquad \text{for } 0 \geq \sigma_A \geq \sigma_B$$

Road maps and important design eßquations for the Stress-Life Method

As stated in Section 6–16, there are three categories of fatigue problems. The important procedures and equations for deterministic stress-life problems are presented here, organized into those three categories.

Completely Reversing Simple Loading

1 Determine S_e' either from test data or

$$S_e' = \begin{cases} 0.5 S_{ut} & S_{ut} \leq 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \qquad (6\text{–}10)$$

2 Modify S_e' to determine S_e.

$$S_e = k_a k_b k_c k_d k_e S_e' \qquad (6\text{–}17)$$

$$k_a = a S_{ut}^b \qquad (6\text{–}18)$$

Table 6–2 Curve Fit Parameters for Surface Factor, Equation (6–18)

Surface Finish	Factor a		Exponent b
	S_{ut}, kpsi	S_{ut}, MPa	
Ground	1.21	1.38	−0.067
Machined or cold-drawn	2.00	3.04	−0.217
Hot-rolled	11.0	38.6	−0.650
As-forged	12.7	54.9	−0.758

Rotating shaft. For bending or torsion,

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.3 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 7.62 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < 254 \text{ mm} \end{cases} \qquad (6\text{–}19)$$

For axial,

$$k_b = 1 \qquad (6\text{–}20)$$

Nonrotating member. For bending, use Table 6–3 for d_e and substitute into Equation (6–19) for d.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}25)$$

$$S_T/S_{RT} = 0.98 + 3.5(10^{-4})T_F - 6.3(10^{-7})T_F^2$$

$$S_T/S_{RT} = 0.99 + 5.9(10^{-4})T_C - 2.1(10^{-6})T_C^2 \qquad (6\text{–}26)$$

Either use the ultimate strength from Equation (6–26) to estimate S_e at the operating temperature, with $k_d = 1$, or use the known S_e at room temperature with $k_d = S_T/S_{RT}$ from Equation (6–26).

Table 6–4 Reliability Factor k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702

3 Determine fatigue stress-concentration factor, K_f or K_{fs}. First, find K_t or K_{ts} from Table A–15.

$$K_f = 1 + q(K_t - 1) \qquad \text{or} \qquad K_{fs} = 1 + q_s(K_{ts} - 1) \qquad (6\text{–}32)$$

Obtain q from either Figure 6–26 or 6–27.

Alternatively,

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \tag{6-34}$$

Bending or axial:

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad 50 \le S_{ut} \le 250 \text{ kpsi}$$

$$\sqrt{a} = 1.24 - 2.25(10^{-3})S_{ut} + 1.60(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \quad 340 \le S_{ut} \le 1700 \text{ MPa} \tag{6-35}$$

Torsion:

$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad 50 \le S_{ut} \le 220 \text{ kpsi}$$

$$\sqrt{a} = 0.958 - 1.83(10^{-3})S_{ut} + 1.43(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \quad 340 \le S_{ut} \le 1500 \text{ MPa} \tag{6-36}$$

4 Apply K_f to the nominal completely reversed stress, $\sigma_a = K_f \sigma_{a0}$.

5 Determine f from Figure 6–23 or Equation (6–11). For S_{ut} lower than the range, use $f = 0.9$.

$$f = 1.06 - 2.8(10^{-3})S_{ut} + 6.9(10^{-6})S_{ut}^2 \quad 70 < S_{ut} < 200 \text{ kpsi}$$

$$f = 1.06 - 4.1(10^{-4})S_{ut} + 1.5(10^{-7})S_{ut}^2 \quad 500 < S_{ut} < 1400 \text{ MPa} \tag{6-11}$$

$$a = (f S_{ut})^2 / S_e \tag{6-13}$$

$$b = -[\log (f S_{ut}/S_e)]/3 \tag{6-14}$$

6 Determine fatigue strength S_f at N cycles, or, N cycles to failure at a reversing stress σ_{ar}.

(*Note:* This only applies to purely reversing stresses where $\sigma_m = 0$.)

$$S_f = aN^b \tag{6-12}$$

$$N = (\sigma_{ar}/a)^{1/b} \tag{6-15}$$

Fluctuating Simple Loading

For S_e, K_f or K_{fs}, see previous subsection.

1 Calculate σ_m and σ_a. Apply K_f to both stresses.

$$\sigma_a = |\sigma_{max} - \sigma_{min}|/2 \qquad \sigma_m = (\sigma_{max} + \sigma_{min})/2 \tag{6-8, 6-9}$$

2 Check for infinite life with a fatigue failure criterion. Use Goodman criterion for conservative result, or another criterion from Section 6–13.

$$\sigma_m \ge 0 \qquad n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1} \tag{6-41}$$

$$\sigma_m < 0 \qquad n_f = \frac{S_e}{\sigma_a} \tag{6-42}$$

3 Check for localized yielding.

$$n_y = \frac{S_y}{\sigma_{max}} = \frac{S_y}{\sigma_a + |\sigma_m|} \tag{6-43}$$

4 For finite-life, find an equivalent completely reversed stress to use on the *S-N* diagram with Equation (6–15). Select one of the following criterion. Discussion of merits is in Section 6–14.

Goodman:
$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/S_{ut}} \qquad\qquad\qquad (6\text{–}59)$$

Morrow:
$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/\tilde{\sigma}_f} \qquad \text{or} \qquad \sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m/\sigma_f'} \qquad (6\text{–}60)$$

Estimate for steel: $\quad \sigma_f' = S_{ut} + 50 \text{ kpsi} \qquad \text{or} \qquad \sigma_f' = S_{ut} + 345 \text{ MPa} \qquad (6\text{–}44)$

SWT:
$$\sigma_{ar} = \sqrt{\sigma_{\max}\sigma_a} = \sqrt{(\sigma_m + \sigma_a)\sigma_a} \qquad\qquad (6\text{–}61)$$

Walker:
$$\sigma_{ar} = \sigma_{\max}^{1-\gamma}\sigma_a^{\gamma} = (\sigma_m + \sigma_a)^{1-\gamma}\sigma_a^{\gamma} \qquad\qquad (6\text{–}62)$$

Estimate for steel: $\quad \gamma = -0.0002S_{ut} + 0.8818 \qquad (S_{ut} \text{ in MPa})$

$$\gamma = -0.0014S_{ut} + 0.8818 \qquad (S_{ut} \text{ in kpsi}) \qquad (6\text{–}57)$$

If determining the finite life *N* with a factor of safety *n*, substitute σ_{ar}/n for σ_{ar} in Equation (6–15). That is,

$$N = \left(\frac{\sigma_{ar}/n}{a}\right)^{1/b}$$

Combination of Loading Modes

See previous subsections for earlier definitions.

1 Calculate von Mises stresses for alternating and mean stress states, σ_a' and σ_m'. When determining S_e, do not use K_c nor divide by K_f or K_{fs}. Apply K_f and/or K_{fs} directly to each specific alternating and mean stress. For the special case of combined bending, torsional shear, and axial stresses

$$\sigma_a' = \{[(K_f)_{\text{bending}}(\sigma_{a0})_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_{a0})_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{a0})_{\text{torsion}}]^2\}^{1/2}$$
$$(6\text{–}66)$$

$$\sigma_m' = \{[(K_f)_{\text{bending}}(\sigma_{m0})_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_{m0})_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{m0})_{\text{torsion}}]^2\}^{1/2}$$
$$(6\text{–}67)$$

2 Apply stresses to fatigue criterion [see previous subsection].

3 To check for yielding, apply the distortion energy theory as usual by putting maximum stresses on a stress element and calculating a von Mises stress.

$$n_y = S_y/\sigma_{\max}'$$

Or for a conservative check, apply the yield line with the von Mises stresses for alternating and mean stresses.

$$n_y = \frac{S_y}{\sigma_a' + \sigma_m'} \qquad\qquad (6\text{–}43)$$