 Name: Solution

INSTRUCTIONS:

This quiz is open-book, open-note, and you may work with your classmates.

GIVEN:

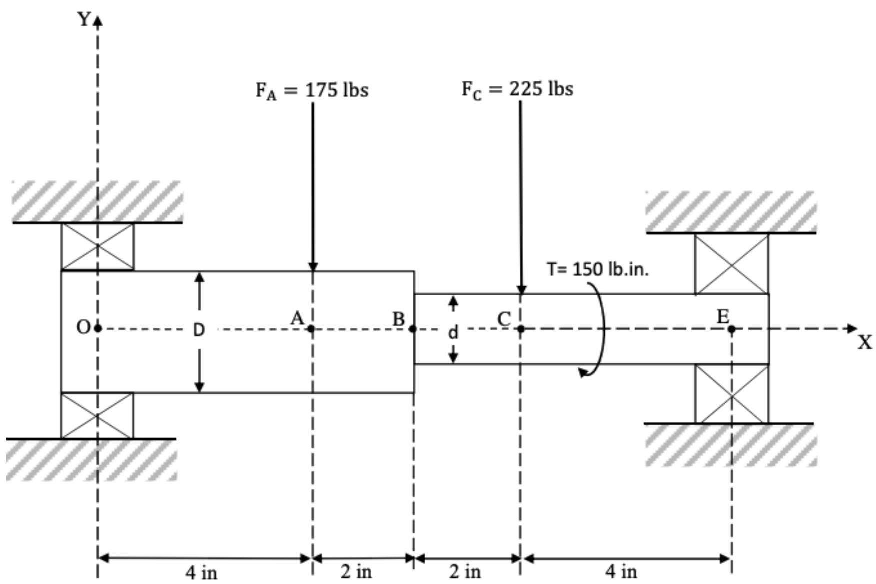
The AISI 1030 hot rolled steel shaft is rotating at a constant speed in the simply supported bearings at points O and E.

The shaft has yield strength $S_y = 37.5$ kpsi, ultimate tensile strength $S_{ut} = 68$ kpsi, and a fully-corrected endurance limit of $S_e = 18.3$ kpsi.

The two constant diameters of the stepped shaft are $D = 2$ in and $d = 1.2$ in.

The constant vertical loads at locations A and C are $F_A = 175$ lbf and $F_C = 225$ lbf and the shaft transmits a constant torque $T = 150$ lbf-in.

The fatigue stress concentration factors at the step are $K_f = 3$ for bending and $K_{fs} = 2.5$ for torsion.



FIND:

a) Sketch diagrams showing the internal loads (bending and torsion) acting on the rotating shaft.

b) Identify the critical cross-section of the shaft.

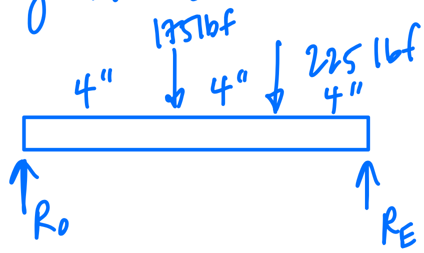
c) For a point on the circumference of the shaft at the critical cross-section, sketch the bending stress as a function of time.

d) For a point on the circumference of the shaft at the critical cross-section, sketch the torsional shear stress as a function of time.

e) The factor of safety for infinite life using the Goodman criterion.

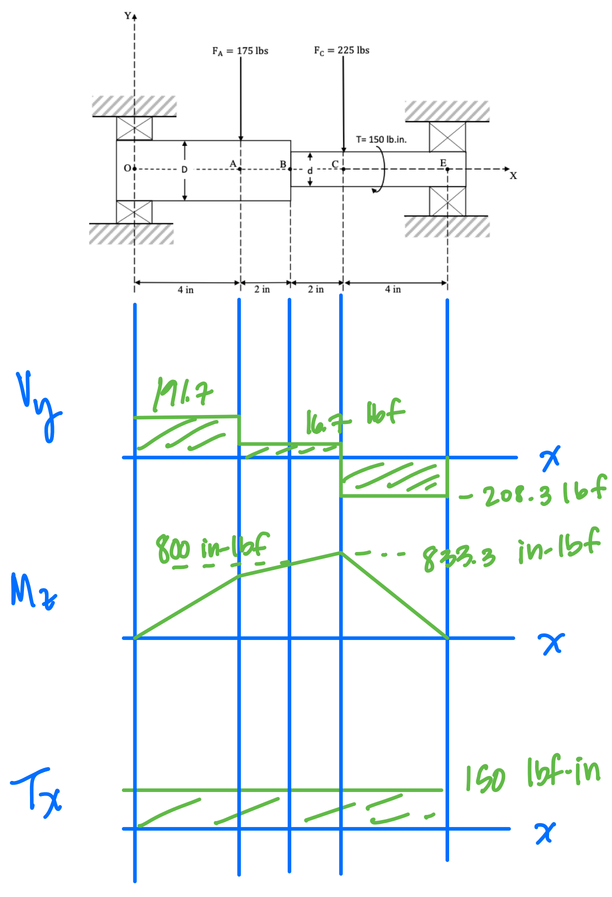
f) The factor of safety for yielding.

a) solving for reactions.



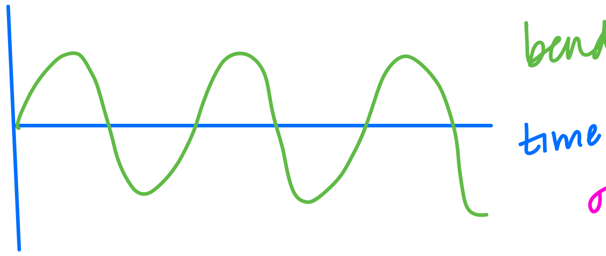
$\Sigma M_0 = 0 \qquad 12 \cdot R_E - 8 \cdot 225 - 4 \cdot 175 = 0 \rightarrow R_E = 208.3 \text{ lbf}$

$\Sigma F_y = 0 \qquad R_0 + R_E - 225 - 175 = 0 \rightarrow R_0 = 400 - 208.3 = 191.7 \text{ lbf}$



b) critical cross-section is at B due to relatively high bending moment and stress raiser.

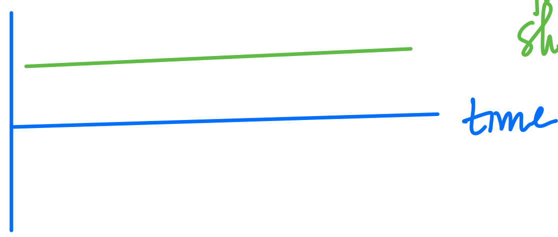
c) σ



bending stress is completely reversed

$\sigma_m = 0 \quad \sigma_a = \sigma_{max}$

d) τ



torsional shear stress is constant

$\tau_a = 0 \quad \tau_m = \tau_{max}$

e) $$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$$

0; no axial load — 0; torque is constant

$$\sigma_a' = \sqrt{\left[K_{f,bend}\,\sigma_{a,bend} + K_{f,axial}\,\cancel{\sigma_{a,axial}}\right]^2 + 3\left[K_{fs}\,\cancel{\tau_a}\right]^2}$$

$$\sigma_a' = K_{f,bend}\,\sigma_{a,bend} = 3 \cdot \frac{Mc}{I} = 3 \cdot \frac{800\text{ in-lbf} \cdot 0.6\text{ in}}{\frac{\pi}{64}(1.2\text{ in})^4} = 14150\text{ psi}$$

0; completely reversed — 0; no axial load

$$\sigma_a' = \sqrt{\left[K_{f,bend}\,\cancel{\sigma_{a,bend}} + K_{f,axial}\,\cancel{\sigma_{a,axial}}\right]^2 + 3\left[K_{fs}\,\tau_a\right]^2}$$

$$\sigma_m' = \sqrt{3}\,K_{fs}\,\tau_a = \sqrt{3} \cdot 2.5 \cdot \frac{Tc}{J} = \sqrt{3} \cdot 2.5 \cdot \frac{150\text{ in-lbf} \cdot 0.6\text{ in}}{\frac{\pi}{32}(1.2\text{ in})^4}$$

$$= 1914\text{ psi}$$

$$\frac{1}{n_f} = \frac{14.15}{18.3} + \frac{1.914}{68} \rightarrow \boxed{n_f = 2.1}$$

f) $$n_y = \frac{S_y}{\sigma_a' + \sigma_m'} = \frac{37.5 \text{ kpsi}}{14.15 + 1.914} = \boxed{2.3 = n_y}$$