October 5, 2022

INSTRUCTIONS

Begin each problem in the space provided.

Write on the front side of the paper only. **Work appearing on the back side of the paper will not be graded.** Extra paper is available in the exam room.

If your solution does not follow a logical thought process, it will be assumed to be in error.

You must turn in your crib sheet with your exam.

PROBLEM No. 1 (25 points)

Problem 1 consists of 10 questions. Each question is worth 2.5 points.

(a) Machine components are often designed with multiple factors of safety.

● True
○ False

each failure mode will have its own fact of safety

(b) Why should factors of safety not be specified beyond one decimal place (e.g., 1.2 instead of 1.2493)

due to the uncertainty in the calculations (load, geometry, material properties all have distributions and are not exact).

(c) Why do machine component failures occur? Select all that apply.

- [x] Inadequate design
- [x] Bad builds (fabrication issues)
- [x] Insufficient maintenance
- [x] Improper or unanticipated use

from the guest speaker on Sept 16

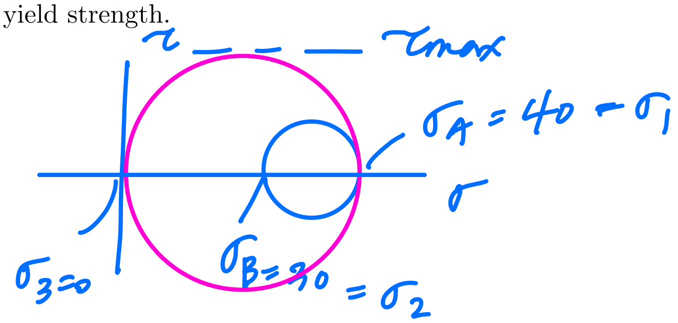
(d) Which approach to fatigue analysis is a damage tolerant approach? One that assumes cracks exist and manages the structure with periodic inspections to detect a crack before it becomes critical?

○ Stress life
○ Strain life
● Linear elastic fracture mechanics

(e) The loading on a machine component was slowly increased until the component yielded. Yielding occurred when the loading produced a state of plane stress with $\sigma_A = 40$ kpsi and $\sigma_B = 30$ kpsi.

Using the MSS failure criterion, find the material's yield strength.

$\boxed{S_y = 40 \text{ kpsi}}$



$$n_y = 1 = \frac{S_y'}{\sigma_1 - \sigma_3}$$

$$S_y' = \sigma_1 - \sigma_3 = 40 - 0 \text{ kpsi}$$

(f) The loading on a machine component was slowly increased until the component yielded. Yielding occurred when the loading produced a state of plane stress with $\sigma_A = 40$ kpsi and $\sigma_B = 30$ kpsi.

Using the DE failure criterion, find the material's yield strength.

$S_y =$ 36.05 kpsi

$n_y = 1 = \frac{S_y'}{\sigma'} \rightarrow S_y = \sigma'$

$$\sigma' = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}} = \sqrt{\frac{(40-30)^2+(30)^2+(40)^2}{2}}$$

(g) A part is subjected to cyclic loading at a frequency of 100 Hz.

The part is required to be designed for infinite life.

Which material should be used?

- [x] AISI 1035 steel
- [ ] A6061 aluminum alloy
- [ ] Either AISI 1035 steel or A6061 aluminum alloy can be used
- [ ] Neither AISI 1035 steel nor A6061 aluminum alloy should be used

Al does not have ∞ life.

(h) A rotating machine component with diameter $d = 40$ mm has a fully corrected endurance limit of $S_e = 340$ MPa and is subjected to torsional loading.

If the diameter was increased to $d = 60$ mm and all other factors remain the same, what is S_e for the component with the larger diameter?

$S_e =$ 323 kpsi

$$S_{e,60mm} = \frac{k_{b,60mm}}{k_{b,40mm}} \cdot S_{e,40mm} = \frac{1.51(60)^{-0.157}}{1.24(40)^{-0.107}} \cdot 340 \text{ MPa} = \frac{0.794}{0.836} \cdot 340 \text{ MPa}$$

(i) Fatigue failures occur due to changes in a part's material properties.

- [ ] True
- [x] False

(j) The stepped shaft shown rotates at a constant speed.
What is the stress concentration factor, K_t, at the location where load F is applied?

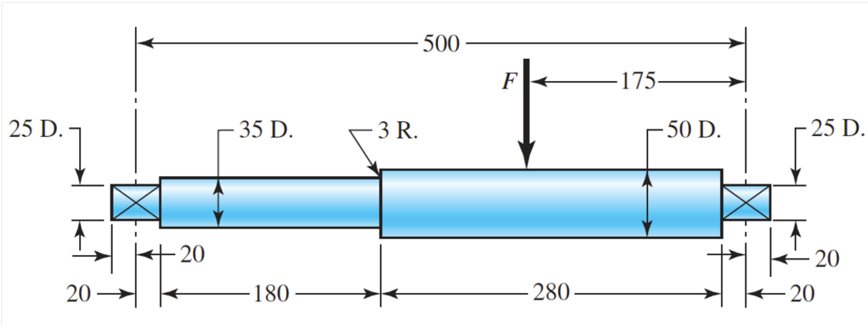
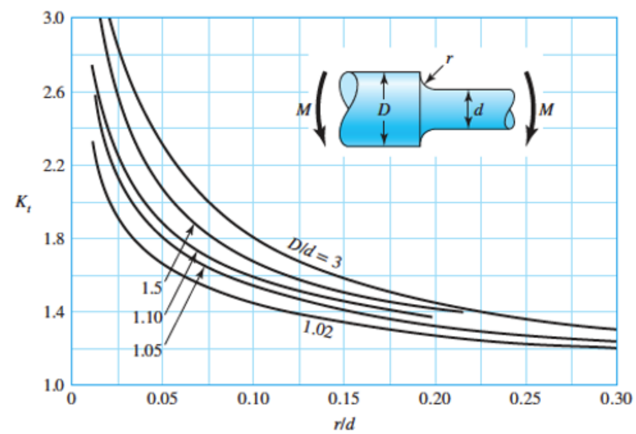


Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



$K_t =$ 1

there is not a stress raiser where F is applied

PROBLEM No. 2 (25 points)

Rod OAB has length $3L$ and diameter $d = L/8$.

The rod is supported by a pin joint at O and by a roller at A.

Axial load P and transverse load $2P$ act at B.

The rod is made of a ductile material with yield strength S_y.

Determine the following.

a) Solve for the reactions at O and A.

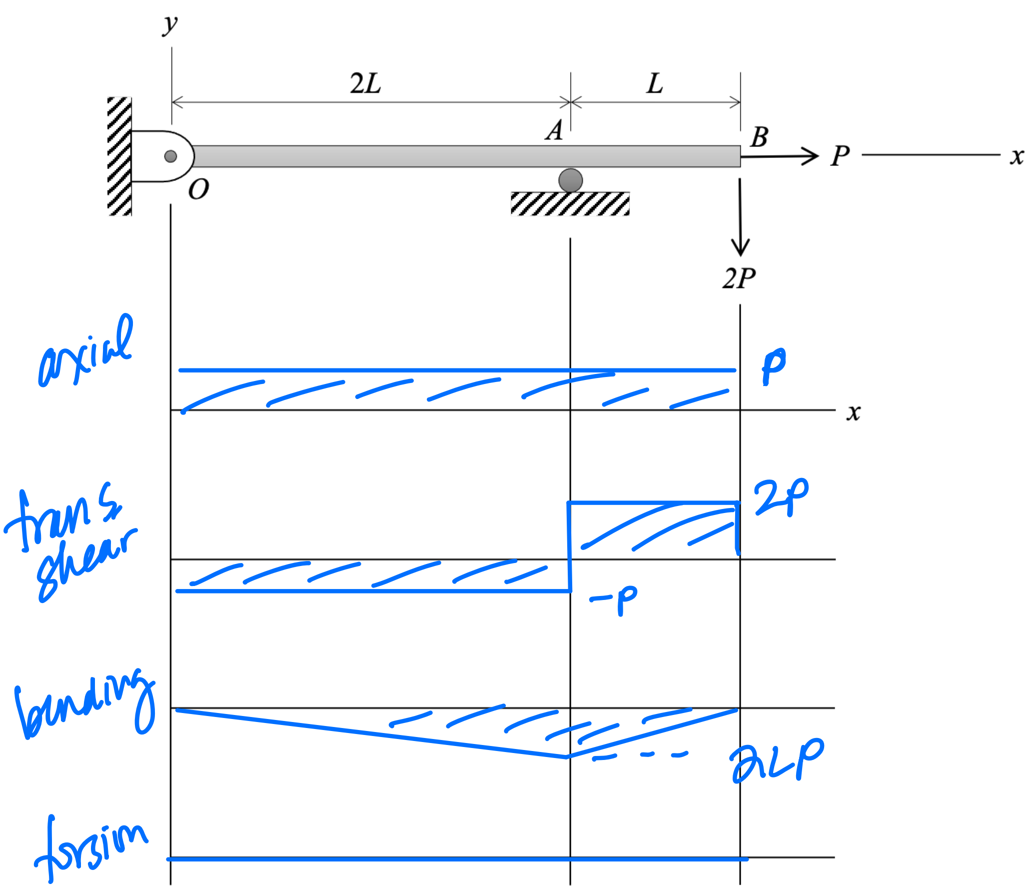
b) Sketch and label diagrams of the internal loads on the axes provided.

c) Identify the critical cross-section of rod OAB.

d) Identify the critical element on the cross-section identified in part (c). You may use the attached Combined Stress Analysis Worksheet to aid your analysis.

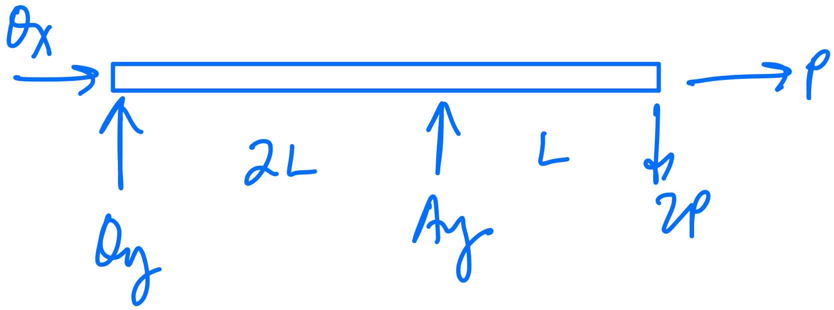
e) Show the state of stress on a stress element for the critical element.

f) The factor of safety for the critical element in terms of variables P, L, and S_y. Use both the distortion energy (DE) and maximum shear stress (MSS) failure theories. If needed, axes to draw Mohr's circle are provided on the next page.



PROBLEM No. 2 (continued)

a)



$\Sigma F_x = 0 \rightarrow O_x = -P$

$\Sigma M_o = 0 \rightarrow \cancel{2}\cancel{L}A_y - \cancel{2}P \cdot 3\cancel{L} = 0 \rightarrow A_y = 3P$

$\Sigma F_y = 0 \rightarrow O_y + A_y - 2P = 0 \rightarrow O_y = -P$

b) see axes provided

c) critical cross-section is just to right of A

d) see worksheet. critical element is on top (+y) of rod.

e) see next page

f) $n_y = \dfrac{S_y}{\sigma'}$ for DE

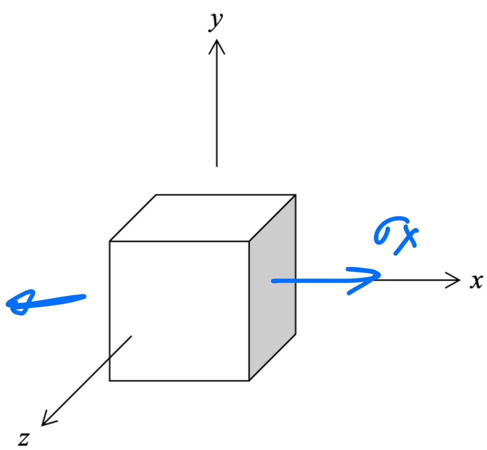
$\sigma' = \sigma_x \rightarrow n_y = \dfrac{S_y L^2}{10500 P}$

same for this case.

$n_y = \dfrac{S_y}{\sigma_1 - \sigma_3}$ $\sigma_1 = \sigma_x$ $\sigma_2 = \sigma_3 = 0 \rightarrow n_y = \dfrac{S_y L^2}{10500 P}$

PROBLEM No. 2 (continued)

Stress element for the critical element:



$$\sigma_x = \frac{P}{A} + \frac{Mc}{I}$$

$$= \frac{P}{\frac{\pi}{4}(L/8)^2} + \frac{2PL\cdot(L/16)}{\frac{\pi}{64}(L/8)^4}$$

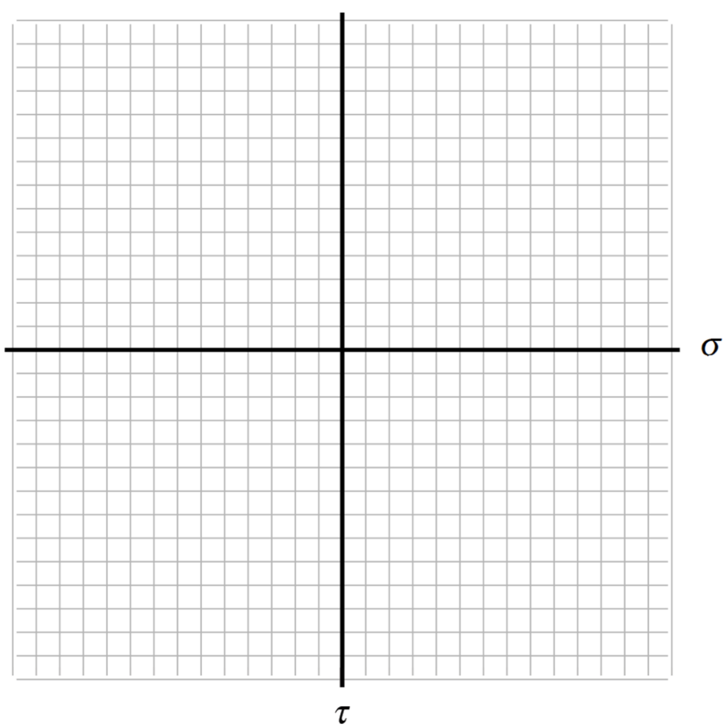
$$= \frac{256P}{\pi L^2} + \frac{8PL^2}{\pi(L/8)^4}$$

$$= \frac{256P}{\pi L^2} + \frac{8^5 P}{\pi L^2}$$

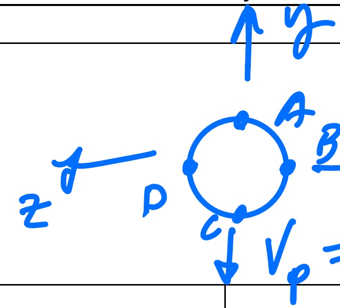
$$= \frac{33024P}{\pi L^2} = 10500\frac{P}{L^2}$$

note: bending dominates.

Axes to draw Mohr's circle:



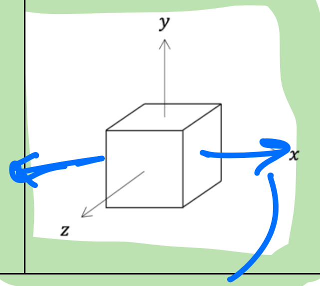
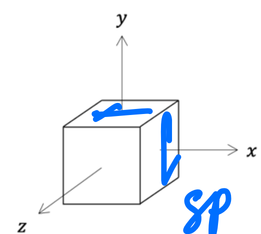
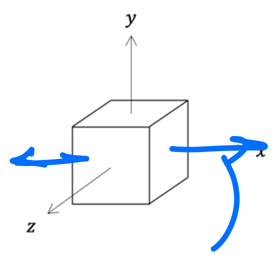
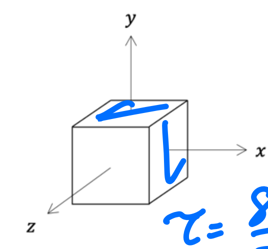
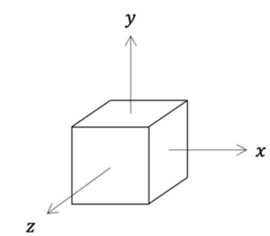
- Draw the machine component's critical cross-section.
- Identify and label the potential locations for the critical element(s) (e.g., top, bottom, right, left, and center)



$M_z = 2PL$

$P_x = P$ out of page

$V_p = 2P$

Potential location of critical element		A	B	C	D	
Internal load	Axial	$\sigma_x = \frac{P}{A}$ →				
	Torsion	none →				
	Transverse shear	0	$\tau_{xy} = -\frac{2P}{A} \cdot \frac{4}{3}$	0	$\tau_{xy} = -\frac{8P}{3A}$	
	Bending	$\sigma_x = \frac{M_z c}{I}$	0	$\sigma_x = -\frac{M_z c}{I}$	0	
Stress element		 $\sigma_x = \frac{P}{A} + \frac{M_z c}{I}$	 $\tau = \frac{8P}{3A}$	 $\sigma_x = \frac{P}{A} - \frac{Mc}{I}$	 $\tau = \frac{8P}{3A}$	

PROBLEM No. 3 (25 points)

The grooved round bar has dimensions $D = 4.6$ mm, $d = 4$ mm, and $r = 0.7$ mm. The bar is not rotating but the axial load cycles from 5 kN in compression to 10 kN in tension. The bar is at room temperature and 90% reliability is desired.

The bar is hot-rolled AISI 1018 steel bar with $S_{ut} = 400$ MPa and $S_y = 220$ MPa. use for k_a

Determine the following.

a) The fully corrected endurance limit, S_e.

b) Draw and label the $S - N$ curve for this part. Use the axes provided.

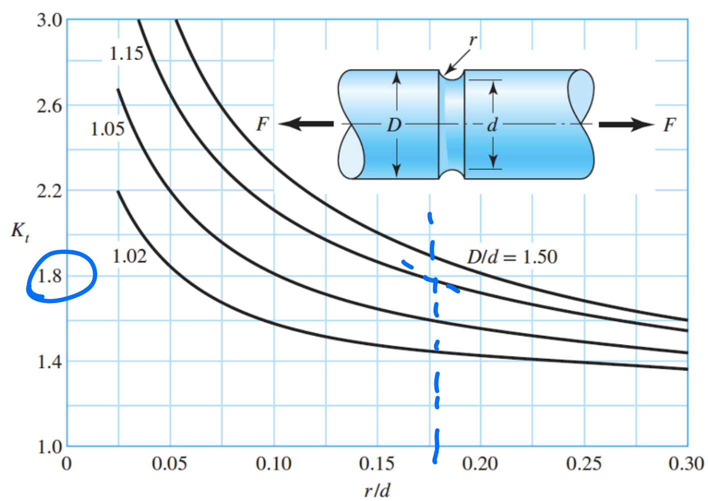
c) The fatigue stress concentration factor K_f.

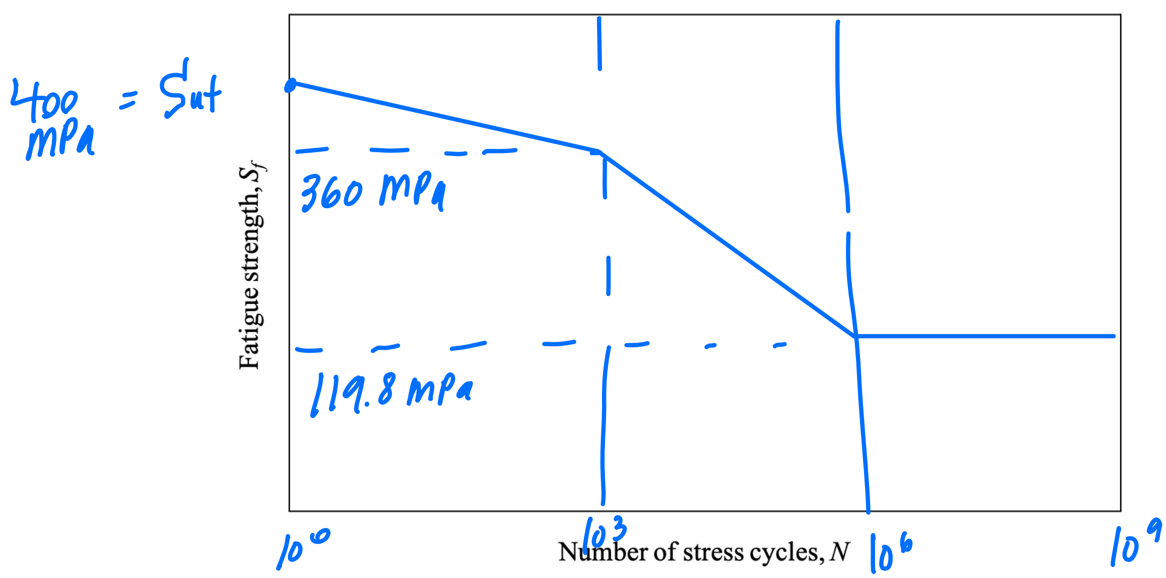
d) The factor of safety for infinite life using the Goodman criterion. If infinite life is not predicted, find the number of cycles until failure.

e) Check for yielding.

Figure A–15–13

Grooved round bar in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.





PROBLEM No. 3 (continued)

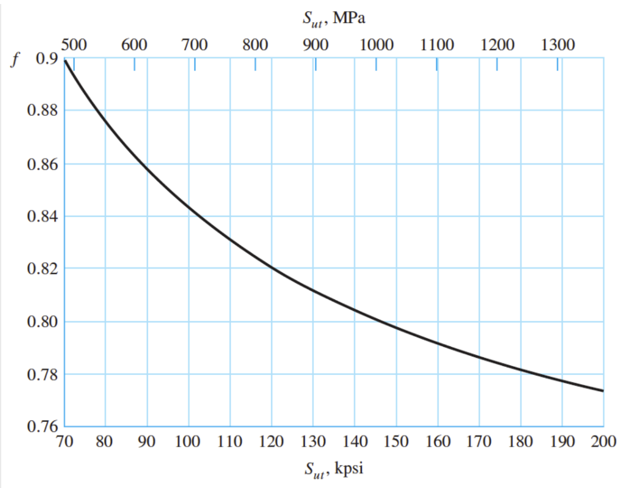


Figure 6–23

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for steels, with $S_e = S_e' = 0.5S_{ut}$ at 10^6 cycles.

$f = 0.9$ for $S_{ut} = 400$ MPa

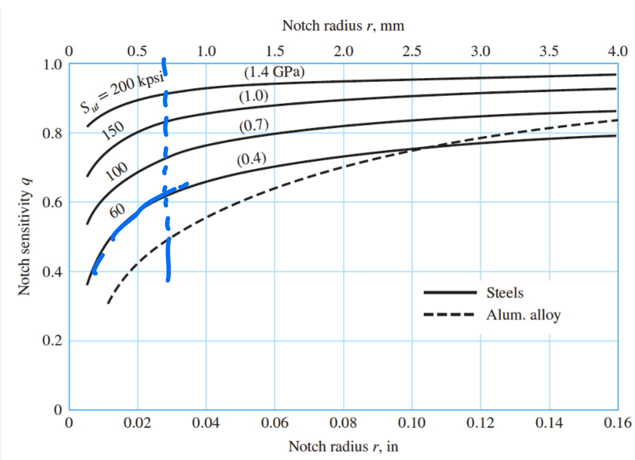


Figure 6–26

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. Source: Sines, George and Waisman, J. L. (eds.), *Metal Fatigue*, McGraw-Hill, New York, 1969.

a) $S_e = k_a k_b k_c k_d k_e S_e' = (0.7857)(1)(0.85)(1)(0.897) \cdot 200$
$= 119.8$ MPa

$S_e' = 0.5\,S_{ut} = 200$ MPa

$k_a = a\,S_{ut}^{\,b} = 38.6(400)^{-0.650} = 0.7857$

$k_b = 1$ for axial load

$k_c = 0.85$ " " "

$k_d = 1$ for room temp

$k_e = 0.897$ for 90% reliability

PROBLEM No. 3 (continued)

b) $\frac{D}{d} = \frac{4.6}{4} = 1.15$

$\frac{r}{d} = \frac{0.7}{4} = 0.175$

$\rightarrow K_t = 1.8$

$K_f = 1 + q(K_t - 1) = 1 + 0.6(1.8-1) = 1.48$

$q = 0.6$ for $r = 0.7\,mm$, $S'_{ut} = 400\,MPa$

d) $\frac{1}{n_f} = \frac{\sigma_a}{S'_e} + \frac{\sigma_m}{S'_{ut}}$

$\sigma_a = K_f \frac{F_a}{\pi d^2/4}$ $\qquad \sigma_m = K_f \frac{F_m}{\pi d^2/4}$

$F_a = \frac{|F_{max} - F_{min}|}{2} = \frac{|2000 - (-1000)|}{2} = 1500\,N$

$F_m = \frac{F_{max} + F_{min}}{2} = \frac{2000 + (-1000)}{2} = 500\,kN$

$\sigma_a = 1.48 \cdot 4 \cdot \frac{1500\,N}{\pi\,(0.004m)^2} = 176.7\,MPa$

$$\sigma_m = \frac{\sigma_a}{3} = 58.9 \text{ MPa}$$

$$\frac{1}{n_f} = \frac{176.7}{119.8} + \frac{58.9}{400} \rightarrow n_f = 0.6$$

$\rightarrow$ finite life

$$\sigma_{ar} = \frac{\sigma_a}{1 - \sigma_m / S_{ut}} = \frac{176.7}{1 - 58.9/400} = 207.2 \text{ MPa}$$

$$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} = \left(\frac{207.2}{1081.8}\right)^{1/-0.1593} = 32100 \text{ cycles}$$

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{(0.9 \cdot 400)^2}{119.8} = 1081.8 \text{ MPa}$$

$$b = -\frac{1}{3} \log \frac{f S_{ut}}{S_e} = -\frac{1}{3} \log \frac{0.9 \cdot 400}{119.8} = -0.1593$$

e) $$n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{220}{176.7 + 58.9} = 0.9$$

however... when K_f is removed for static loading

$$n_y = \frac{S_y}{\frac{1}{1.48}(\sigma_a + \sigma_m)} = 1.3 \rightarrow \text{yielding is } \underline{\text{not}} \text{ predicted}$$

PROBLEM No. 4 (25 points)

A rotating steel shaft is simply supported by bearings at A and C. The diameter of the larger section is 1.5 inches and the diameter of the smaller section is 1 inch.

The shaft is loaded with constant transverse force of P=300 lbf at point B, and a torque that alternates between T_{min} = -100 lbf-in and T_{max} = +700 lbf-in in section BC.

The shaft has the following material properties: S_{ut} = 100 kpsi, S_y = 62 kpsi, S_e = 30 kpsi (fully corrected).

The fatigue stress concentration factors at the step are K_f = 2.1 for bending and K_{fs} = 2.5 for torsion.

Determine the following.

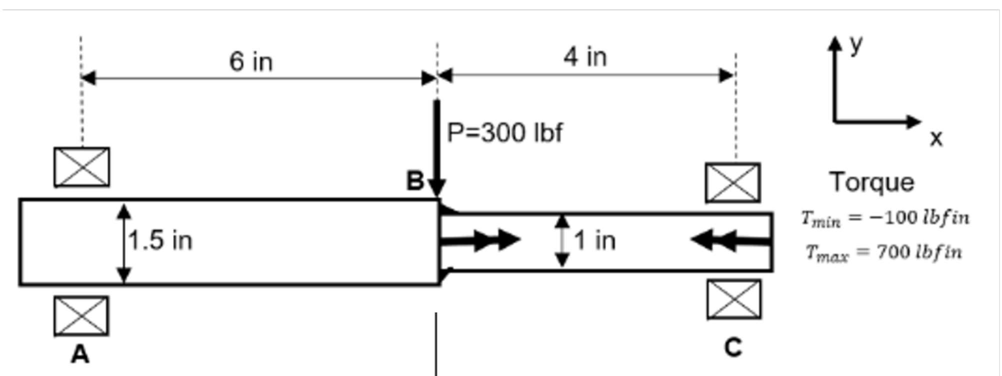
a) Solve for the reactions at A and C.

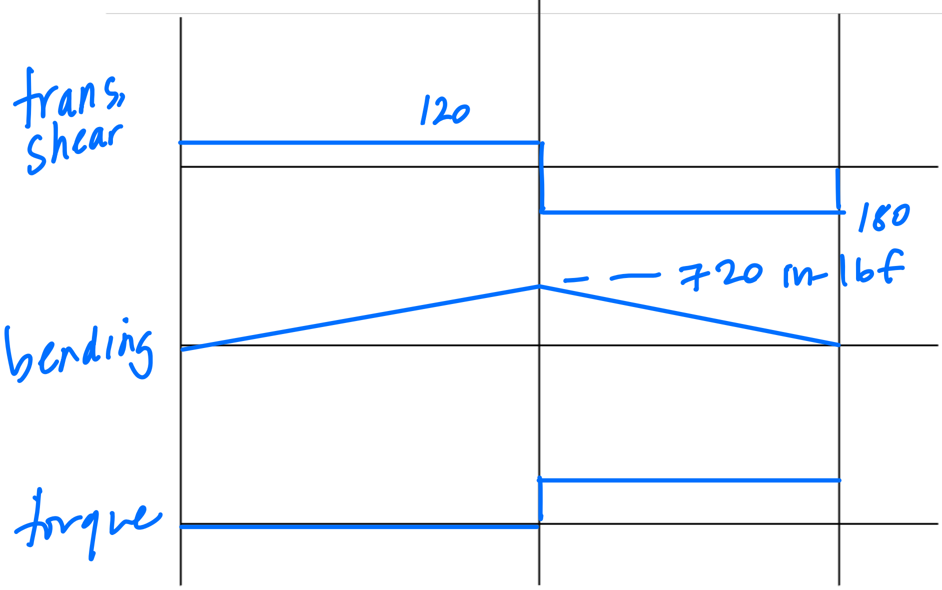
b) Sketch and label diagrams of the internal loads on the axes provided.

c) Identify the critical cross-section of the shaft.

d) The factor of safety for infinite life using the Goodman criterion. If infinite life is not predicted, find the number of cycles until failure.

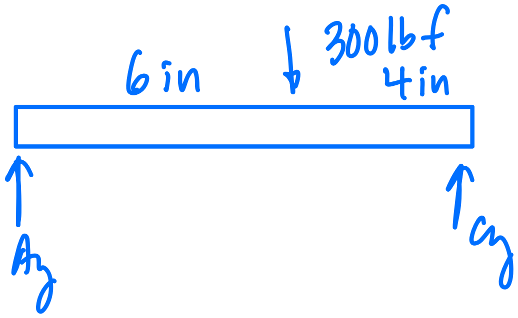
e) Check for yielding.





PROBLEM No. 4 (continued)

a)



$\Sigma M_A = 0 \rightarrow -6 \cdot 300 + 10 \cdot C_y = 0 \rightarrow C_y = 180 \text{ lbf}$

$\Sigma F_y = 0 \rightarrow A_y + C_y - 300 = 0 \rightarrow A_y = 120 \text{ lbf}$

b) see axes

c) critical cross-section is just to right of ~~A~~ B.

$M_a = 720 \text{ lbf·in}$

$M_m = 0$ (Rotating shaft)

$T_a = 400 \text{ lbf·in}$, $T_m = 300 \text{ lbf·in}$

d) $\dfrac{1}{n_f} = \dfrac{\sigma_a'}{S_e} + \dfrac{\sigma_m'}{S_{ut}}$

$$\sigma_a' = \sqrt{\left[2.1\left(\frac{720(0.5)}{\frac{\pi 1^4}{64}}\right)\right]^2 + 3\left[(2.5)\frac{400(0.5)}{\frac{\pi 1^4}{32}}\right]^2} \cong 17.75 \text{ ksi}$$

$$\sigma_m' = \sqrt{3\left[(2.5)\frac{300(0.5)}{\frac{\pi 1^4}{32}}\right]^2} \cong 6.62 \text{ ksi}$$

$$n_f = \left(\frac{17.75}{30} + \frac{6.62}{100}\right)^{-1} = 1.5$$

combined bending, torsional shear, and axial stresses

No axial

$$\sigma_a' = \{[(K_f)_{\text{bending}}(\sigma_{a0})_{\text{bending}} + \cancel{(K_f)_{\text{axial}}(\sigma_{a0})_{\text{axial}}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{a0})_{\text{torsion}}]^2\}^{1/2} \quad (6\text{–}66)$$

$\sigma_m = 0$

$$\sigma_m' = \{[\cancel{(K_f)_{\text{bending}}(\sigma_{m0})_{\text{bending}}} + \cancel{(K_f)_{\text{axial}}(\sigma_{m0})_{\text{axial}}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{m0})_{\text{torsion}}]^2\}^{1/2} \quad (6\text{–}67)$$

PROBLEM No. 4 (continued)

e-) $n_y = \frac{S_y}{\sigma_a' + \sigma_m'} \Rightarrow n_y = \frac{62}{(17.75) + (6.62)} = 2.5$