ME 354 Fall 2022: Quiz 5 Name:

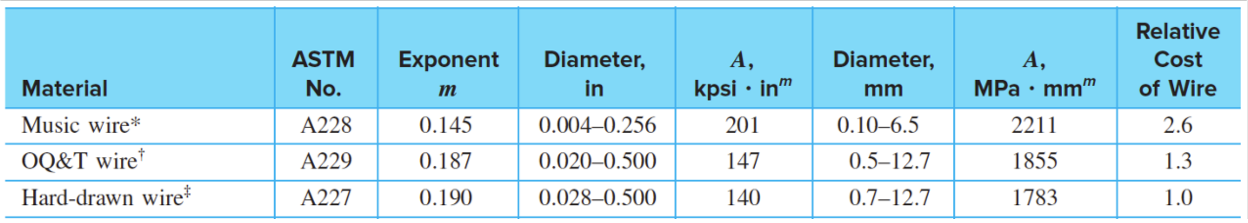
INSTRUCTIONS:

This quiz is open-book, open-note, and you may work with your classmates.

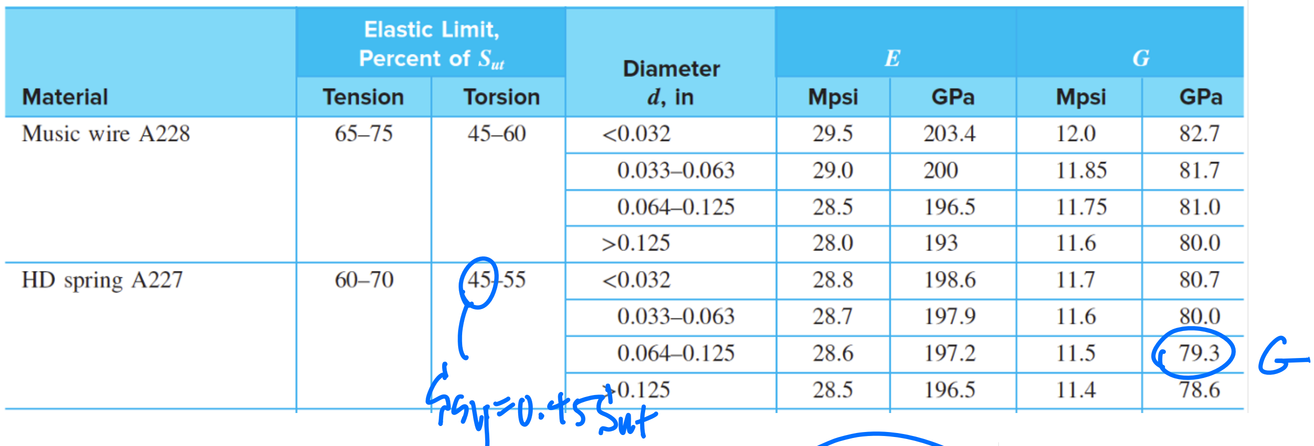
GIVEN:

A helical compression spring is wound from 2-mm ASTM A227 hard-drawn wire and has an outside diameter of 19 mm.

The spring ends are closed and ground, and there are ten total coils. Note that "closed" and "squared" are synonyms.



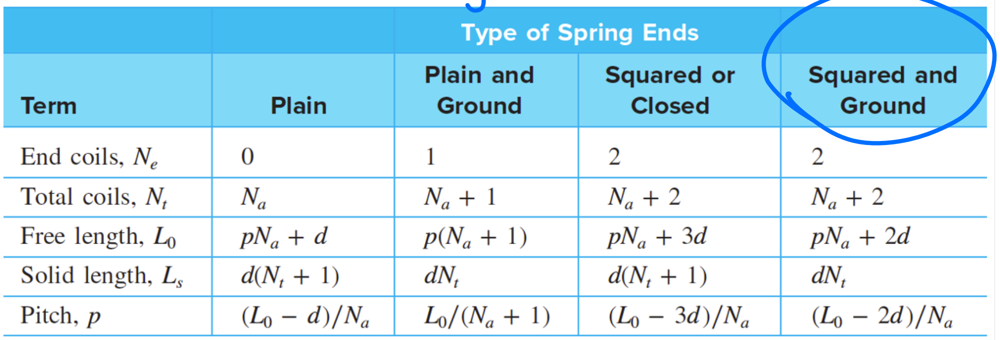
Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0



Material	Elastic Limit, Percent of S_{ut}		Diameter d, in	E		G	
	Tension	Torsion		Mpsi	GPa	Mpsi	GPa
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60–70	45–55	<0.032	28.8	198.6	11.7	80.7
			0.033–0.063	28.7	197.9	11.6	80.0
			0.064–0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6

$S_{sy} = 0.45 S_{ut}$

G



Term	Plain	Plain and Ground	Squared or Closed	Squared and Ground
		Type of Spring Ends		
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

FIND:

a) The torsional yield strength of the wire, S_{sy}.

b) The spring rate, k.

c) The static load that can be applied before the spring yields.

d) The free length of the spring, L_0, for the load found in part (c) to cause the spring to become solid.

a) $S'_{sy} = 0.45 S_{ut} = 0.45 \frac{A}{d^m} = 0.45 \frac{1783}{2^{0.19}} = 703.3 \text{ MPa}$

b) $k \approx \frac{d^4 G}{8 D^3 N_a} = \frac{(0.002\,m)^4 \cdot 79.3 \cdot 10^9\,N/m^2}{8\,(0.017\,m)^3 \cdot 8} = 4035\,N/m$

$d = 2\,mm = 0.078\,in$

$G = 79.3\,GPa$

$D = OD - d = 19 - 2 = 17\,mm$

$N_a = N_t - 2 = 8$

c) $S'_{sy} = \tau = K_B \frac{8FD}{\pi d^3}$

$K_B = \frac{4C+2}{4C-3} = 1.16$

$C = \frac{D}{d} = \frac{17}{2} = 8.5$

$$F = \frac{S'_{sy}\,\pi d^3}{8 K_B D} = \frac{703.3 \cdot 10^6\,N/m^2 \cdot \pi\,(0.002\,m)^3}{8 \cdot 1.16 \cdot (0.017\,m)}$$

$= 111.9\,N$

d) $k = \frac{\overset{\text{part (c)}}{F}}{L_0 - L_s} = \overset{\text{from part (b)}}{4035\,N/m} \rightarrow L_0 = \frac{111.9\,N}{4035\,N/m} + L_s = 47.7\,mm$

$L_s = d N_t = 2\,mm \cdot 10 = 20\,mm$