November 9, 2022

INSTRUCTIONS

Begin each problem in the space provided.

Write on the front side of the paper only. **Work appearing on the back side of the paper will not be graded.** Extra paper is available in the exam room.

If your solution does not follow a logical thought process, it will be assumed to be in error.

You must turn in your crib sheet with your exam.

In which section are you enrolled?

◯ Hess - 10:30-11:20 am

◯ Hess - 12:30-1:20 pm

◯ Akin - 3:30-4:20 pm

PROBLEM No. 1 (25 points)

Problem 1 consists of 10 questions. Each question is worth 2.5 points.

(a) SAE 1040 steel is what percent carbon (C) by weight?

- ◯ 1%
- ◯ 4%
- ● 0.4%
- ◯ 0.04%

(b) SAE 1040 is a high-carbon steel.

- ◯ True
- ● False

(c) Which type of steel is most appropriate for making knives and punches?

- ◯ Low-carbon steel
- ◯ Medium-carbon steel
- ◯ High-carbon steel
- ● Ultra-high carbon steel

(d) What are the typical properties of cold working (CD) process? Select all that apply.

- [x] Applied at room temperature
- [x] Applicable for I-beams
- [ ] Poor surface finish
- [x] Requires less machining
- [x] Higher production cost than hot-rolling
- [x] Better resulting tolerances than hot-rolling

(e) Which properties of steel can be modified with heat treatment? Select all that apply.

- [x] Strength
- [x] Ductility
- [ ] Modulus of elasticity
- [x] Toughness
- [x] Crystalline structure

(f) A steel member has a Brinell of $H_B = 350$. Estimate the ultimate tensile strength of the steel in kpsi.

$\boxed{S_{ut} = 175 \text{ kpsi}}$

$S_{ut} = 0.5 H_B = 175$ kpsi

(g) Which of the following characteristics are NOT associated with an ideal spring material? Select all that apply.

- ☐ High ultimate strength
- ☐ High yield strength
- ☒ High modulus of elasticity

(h) What type of lubrication occurs in sliding bearings when surfaces physically contact and adhesive or abrasive wear may occur?

- ○ Mixed-film lubrication
- ● Boundary lubrication
- ○ Full-film lubrication

(i) What type of lubrication occurs when the contacting surfaces are non-conforming (e.g., gear teeth)?

- ○ Hydrodynamic lubrication
- ● Elastohydrodynamic (EHD) lubrication
- ○ Hydrostatic lubrication

(j) A rolling element bearing fails when which of the following occurs?

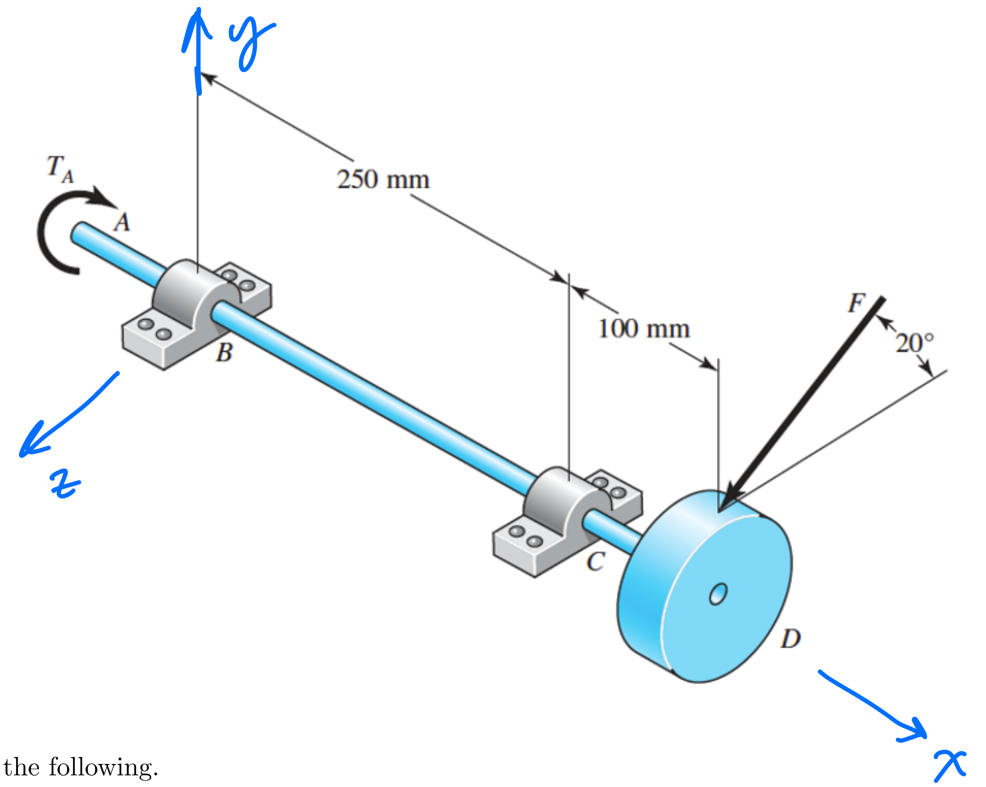
- ○ Crack initiation
- ● Pitting
- ○ Flaking
- ○ Spalling
- ○ Disintegration

PROBLEM No. 2 (15 points)

The 25-mm diameter rotating solid steel shaft is simply supported by rolling element bearings at B and C and driven by a gear (not shown) that meshes with the spur gear at D which has a 150-mm pitch diameter.

The force F from the drive gear acts at a pressure angle of 20° and the shaft rotates at a constant speed. $\rightarrow \vec{\alpha} =$

The spur gear at D is attached to the rotating shaft with a 6-mm square key. The key is 10 mm long and oriented such that the key length is parallel with the shaft. The key is made of 1018 CD steel with $S_y = 370$ MPa and $S_{ut} = 440$ MPa.



Determine the following.

a) An expression for T_A, the torque acting on the shaft, in terms of unknown force F.

b) The maximum force F that can be applied without shearing the key. Use a factor of safety of $n = 1.2$.

c) The maximum force F that can be applied without crushing the key. Use a factor of safety of $n = 1.2$.

d) The rolling element bearings at B and C are attached to the rotating shaft with a shrink fit, requiring a H7/s6 medium drive fit. For a nominal size of 25-mm, what are the the minimum and maximum diameters of the rotating shaft to achieve this fit (d_{min} and d_{max})? Reference tables are provided on the next page.

a) $\Sigma M_x = 0 \qquad F\cos 20 \cdot 75\text{mm} - T_A = 0$

$T_A = F\cos 20 \cdot 75\text{mm}$

b) $n = \dfrac{0.577 S_y}{\tau} \qquad \tau = \dfrac{T}{rwl}$

$$n = \frac{0.577\, S_y \cdot rwl}{F\cos 20 \cdot 75\,mm} \rightarrow F = \frac{0.577\, S_y\, rwl}{n\cos 20 \cdot 75\,mm}$$

$$F = \frac{0.577 \cdot 370\,MPa \cdot 12.5\,mm \cdot 6\,mm \cdot 10\,\cancel{mm}}{1.2 \cdot \cos 20 \cdot 75\,\cancel{mm}} \cdot 10^6\,N/m^2/MPa \cdot \left(\frac{1m}{1000mm}\right)^2$$

$$= 1893\ N$$

c) $n = \frac{S_y}{\sigma}$ $\quad \sigma = \frac{2T}{rhl}$

$$n = \frac{S_y \cdot rhl}{2 \cdot F\cos 20 \cdot 75\,mm} \rightarrow F = \frac{S_y \cdot rhl}{2n\cos 20 \cdot 75\,mm}$$

$$F = \frac{370\,MPa \cdot 12.5\,mm \cdot 6\,mm \cdot 10\,mm}{2 \cdot 1.2 \cdot \cos 20 \cdot 75\,mm} \cdot \frac{10^6\,N/m^2}{MPa} \cdot \left(\frac{1\,m}{1000\,mm}\right)^2$$

$$= 1640\ N$$

d) $d_{max} = d + \delta_F + \Delta d = 25.035 + 0.013 = 25.048\ mm$

$d_{min} = d + \delta_F = 25 + 0.035 = 25.035\ mm$

PROBLEM No. 2 (continued)

Table A–11 A Selection of International Tolerance Grades—Metric Series

(Size Ranges Are for *Over* the Lower Limit and *Including* the Upper Limit. All Values Are in Millimeters)

Basic Sizes	Tolerance Grades					
	IT6	IT7	IT8	IT9	IT10	IT11
0–3	0.006	0.010	0.014	0.025	0.040	0.060
3–6	0.008	0.012	0.018	0.030	0.048	0.075
6–10	0.009	0.015	0.022	0.036	0.058	0.090
10–18	0.011	0.018	0.027	0.043	0.070	0.110
18–30	0.013	0.021	0.033	0.052	0.084	0.130
30–50	0.016	0.025	0.039	0.062	0.100	0.160

$\Delta d = 0.013$

Table A–12 Fundamental Deviations for Shafts—Metric Series

(Size Ranges Are for *Over* the Lower Limit and *Including* the Upper Limit. All Values Are in Millimeters)

Basic Sizes	Upper-Deviation Letter					Lower-Deviation Letter				
	c	d	f	g	h	k	n	p	s	u
0–3	–0.060	–0.020	–0.006	–0.002	0	0	+0.004	+0.006	+0.014	+0.018
3–6	–0.070	–0.030	–0.010	–0.004	0	+0.001	+0.008	+0.012	+0.019	+0.023
6–10	–0.080	–0.040	–0.013	–0.005	0	+0.001	+0.010	+0.015	+0.023	+0.028
10–14	–0.095	–0.050	–0.016	–0.006	0	+0.001	+0.012	+0.018	+0.028	+0.033
14–18	–0.095	–0.050	–0.016	–0.006	0	+0.001	+0.012	+0.018	+0.028	+0.033
18–24	–0.110	–0.065	–0.020	–0.007	0	+0.002	+0.015	+0.022	+0.035	+0.041
24–30	–0.110	–0.065	–0.020	–0.007	0	+0.002	+0.015	+0.022	+0.035	+0.048
30–40	–0.120	–0.080	–0.025	–0.009	0	+0.002	+0.017	+0.026	+0.043	+0.060
40–50	–0.130	–0.080	–0.025	–0.009	0	+0.002	+0.017	+0.026	+0.043	+0.070

$\delta_F = 0.035$

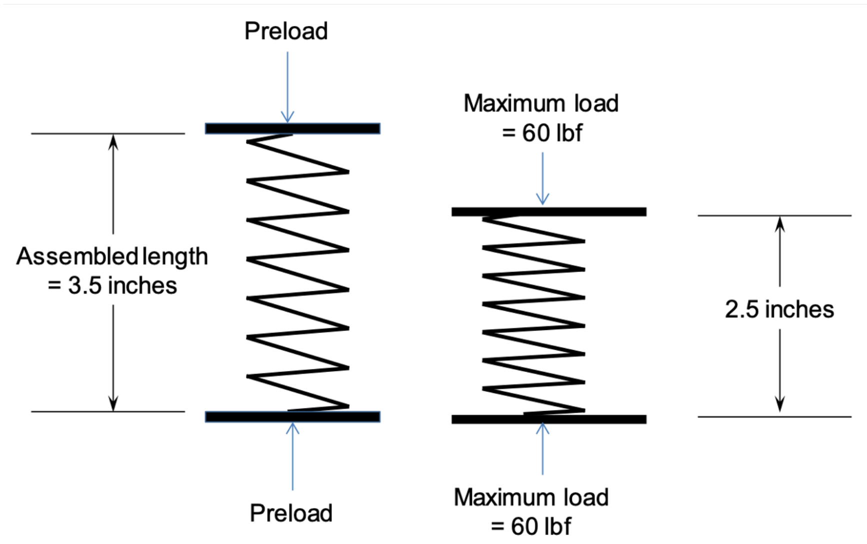
PROBLEM No. 3 (30 points)

A spring application requires a helical compression spring with solid length $L_s = 1.2$ inches and free length $L_0 = 4.0$ inches. The assembled length is $L_a = 3.5$ inches.

The spring has 14 total coils and the ends are squared and ground.

The spring material is music wire (A228) and the spring is peened.

The spring is repeatedly compressed from L_a to a length of $L_m = 2.5$ inches. The force required to compress the spring to L_m is 60 lbf.



Determine the following.

a) The wire diameter d.

b) The stiffness k.

c) The mean coil diameter D and the spring index C.

d) The endurance limit of the spring wire, S_{se}, using the Zimmerli data for the Goodman criterion.

e) The mid-range (τ_m) and alternating (τ_a) components of shear stress for the spring loaded between L_a and L_m.

f) The factor of safety for infinite life using the Goodman failure criterion for the spring loaded between L_a and L_m.

a) $L_s = dN_t \rightarrow d = L_s/N_t = 1.2\text{ in}/14 = 0.0857\text{ in}$

b) $k = \frac{F}{y} = \frac{60\text{ lbf}}{L_0 - L_m} = \frac{60\text{ lbf}}{4-2.5\text{ in}} = 40\text{ lbf/in}$

c) $k \approx \frac{d^4 G}{8D^3 N_a}$

$$D = \left[\frac{d^4 G}{8 N_a k}\right]^{1/3} = \left[\frac{(0.0857\,in)^4 \cdot 11.75 \cdot 10^6\ lbf/in^2}{8 \cdot 12 \cdot 40\ lbf/in}\right]$$

$$N_a = N_t - 2 = 12$$

$$G = 11.75\ Mpsi$$

$$D = 0.5486\ in$$

$$C = \frac{D}{d} = \frac{0.5486\,in}{0.0857\,in} = 6.4$$

d) $$S_{se} = \frac{S_{sa}}{1 - \frac{S_{sm}}{S_{su}}} = \frac{57.5}{1 - \frac{77.5}{192.3}} = 96.3\ kpsi$$

$$S_{sa} = 57.5\ kpsi \qquad S_{sm} = 77.5\ kpsi$$

$$S_{su} = 0.67\, S_{ut} = 192.3\ kpsi$$

$$S_{ut} = \frac{A}{d^m} = \frac{201}{(0.0857)^{0.145}} = 287\ kpsi$$

e) $$\tau_a = K_B \frac{8 F_a D}{\pi d^3} = 1.221 \frac{8 \cdot 20\ lbf \cdot 0.5486\ in}{\pi \cdot (0.0857\,in)^3} = 54.2\ kpsi$$

$$\tau_m = K_B \frac{8 F_m D}{\pi d^3} = 2\tau_a = 108.4\ kpsi$$

$$F_a = \frac{F_{max} - F_{min}}{2} = \frac{60-20}{2} = 20 \text{ lbf}$$

$$F_m = \frac{F_{max} + F_{min}}{2} = \frac{60+20}{2} = 40 \text{ lbf}$$

$F_{max} = 60$ lbf (the force to compress to L_m)

$$F_{min} = k(L_o - L_a) = 40 \frac{\text{lbf}}{\text{in}} (4\text{in} - 3.5\text{in}) = 20 \text{ lbf}$$

$$K_B = \frac{4C+2}{4C-3} = \frac{4(6.4)+2}{4(6.4)-3} = 1.221$$

f) $$\frac{1}{n_f} = \frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}$$

$$\frac{1}{n_f} = \frac{54.2}{96.3} + \frac{108.4}{192.3}$$

$n_f = 0.9 \rightarrow \infty$ life not predicted.

PROBLEM No. 3 (continued)

Table 10–1 Formulas for the Dimensional Characteristics of Compression Springs (N_a = Number of Active Coils)

	Type of Spring Ends			
Term	**Plain**	**Plain and Ground**	**Squared or Closed**	**Squared and Ground**
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Table 10–4 Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6

Table 10–5 Mechanical Properties of Some Spring Wires

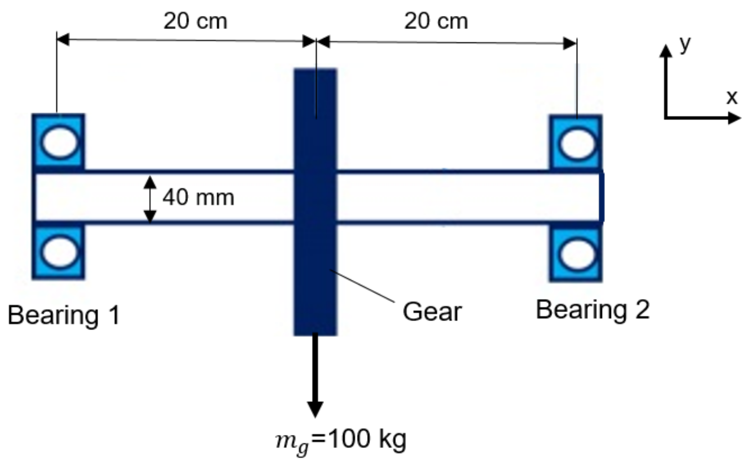
	Elastic Limit, Percent of S_{ut}			E		G	
Material	**Tension**	**Torsion**	**Diameter d, in**	**Mpsi**	**GPa**	**Mpsi**	**GPa**
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0

PROBLEM No. 4 (15 points)

Two identical ball bearings support a 40-mm diameter rotating shaft and a gear.

The bearings are symmetrically assembled on the shaft. The gear has total mass of 100 kg. The weight of the shaft and bearings can be neglected.

See the next page for catalog data, where the catalog rating life is 10^6 cycles.



Determine the following.

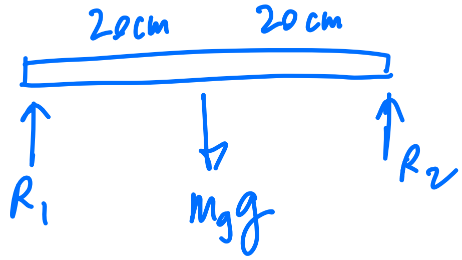
a) The radial load acting at the bearings.

b) Choose a bearing from the catalog based on the dimensions given.

c) For the bearing chosen in part (b), determine the bearing life if 95% reliability is required.

d) Repeat part (c) if the bearing must support a 1 kN axial load in addition to the radial load.

a)



by symmetry $R_1 = R_2 = \frac{m_g g}{2}$

radial load = 490 N

b) see table – choose bearing w/ 40 mm bore to match shaft diameter.

c) $a_1 F_R L_R^{1/a} = F_D L_D^{1/a}$

$a_1 = 0.64$ for 95% reliability

$F_R = 12.6$ kN from catalog

$L_R = 10^6$ cycles

$a = 3$ for ball bearing

$F_D = 490$ N

$$L_D = \left(\frac{a_1 F_R}{F_D}\right)^a L_R = \left(\frac{0.64 \cdot 12600}{490}\right)^3 \cdot 10^6 = 4.5 \cdot 10^9 \text{ cycles}$$

d) $F_e = X_i V F_r + Y_i F_a = 0.56 \cdot 1 \cdot 490 + 1.45 \cdot 1000 = 1724$ N

$$\frac{F_a}{C_0} = \frac{1000 \text{ N}}{9700 \text{ N}} = 0.103$$

$e = 0.3$

$$\frac{F_a}{V F_r} = \frac{1000 \text{ N}}{1 \cdot 490 \text{ N}} = 2.04 > 0.3 \rightarrow i = 2$$

inner ring rotates by inspection

$$L_D = \left(\frac{a_1 F_R}{F_D}\right)^a L_R = \left(\frac{0.64 \cdot 12600}{1724}\right)^3 \cdot 10^6 = 102 \cdot 10^6 \text{ cycles}$$

NARROW 16000 SERIES

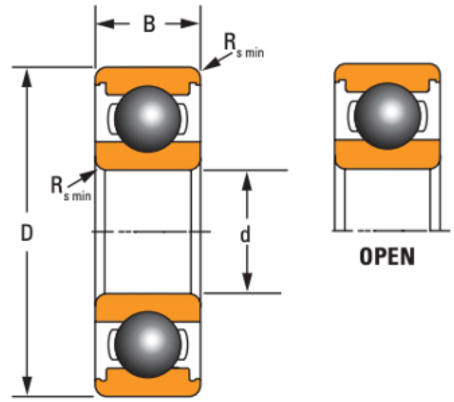


TABLE 3. 16000 SERIES

Bearing No.			Boundary Dimensions				Load Ratings	
Description	Features		Bore d	O.D. D	Width B	Radius $R_{s\ min}$	Dynamic C_r	Static C_{0r}
	ZZ	2RS	mm	mm	mm	mm	kN	kN
16100	•		10	28	8	0.3	4.60	2.00
16101	•	•	12	30	8	0.3	5.10	2.40
16002	•		15	32	8	0.3	5.60	2.80
16003	•		17	35	8	0.3	6.00	3.30
16004			20	42	8	0.3	6.30	3.80
16005	•		25	47	8	0.3	7.00	4.60
16006			30	55	9	0.3	9.20	6.30
16007			35	62	9	0.3	12.20	8.80
16008			40	68	9	0.3	12.60	9.70

$C_{10} = F_R$ C_o

part 6

Table 11–1 Equivalent Radial Load Factors for Ball Bearings

		$F_a/(VF_r) \le e$		$F_a/(VF_r) > e$	
F_a/C_0	e	X_1	Y_1	X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

*Use 0.014 if $F_a/C_0 < 0.014$.

close enough to 0.103

PROBLEM No. 5 (15 points)

A full journal bearing is 28 mm long.

The shaft journal has a diameter of 56 mm with a unilateral tolerance of -0.012 mm (i.e., the journal diameter ranges between 55.988 and 56 mm).

The bearing bore has a diameter of 56.05 mm with a unilateral tolerance of +0.012 mm.

The shaft supports a radial load of 2.4 kN and the journal speed is 1100 rpm.

The lubricant is SAE 40 and the operating temperature is 40°C.

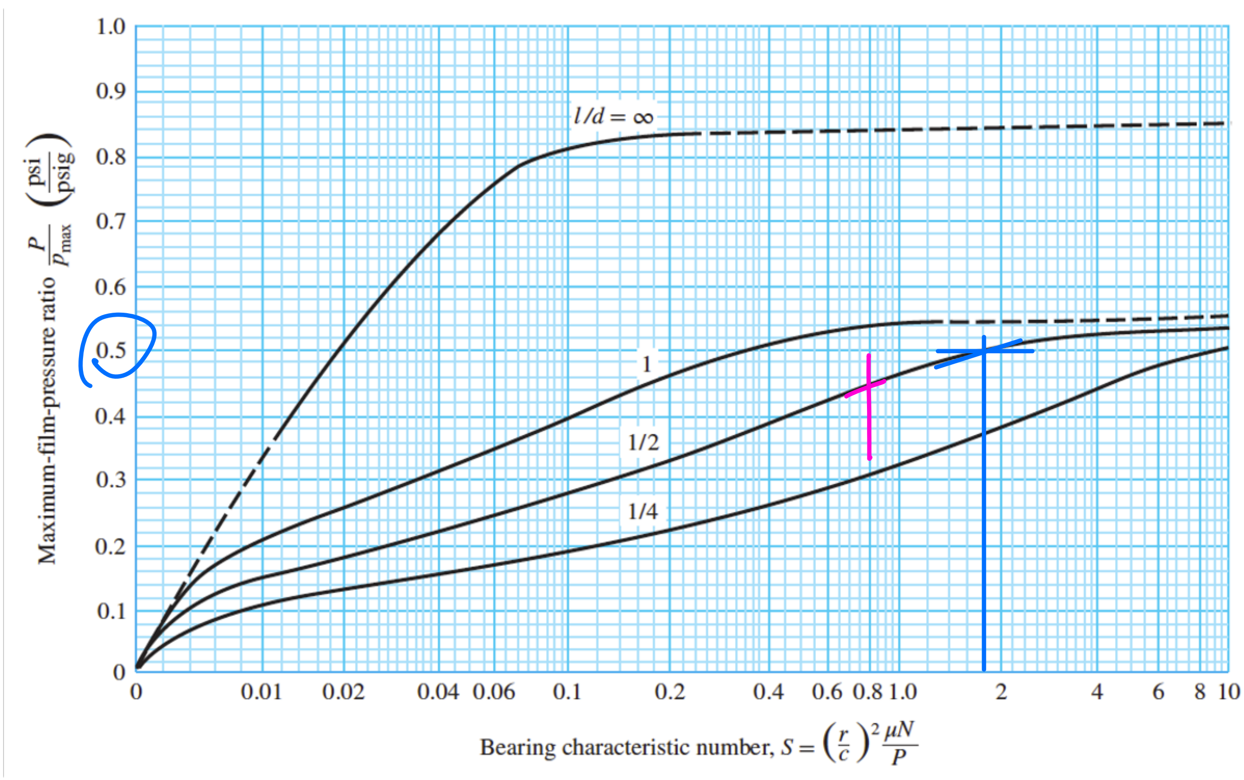
Determine the following.

a) The minimum and maximum clearances.

b) The Sommerfeld number for the minimum clearance assembly, S.

c) The maximum film pressure for the minimum clearance assembly.

d) Will the maximum film pressure increase or decrease if the the bearing is assembled at its maximum clearance? Briefly justify your answer.



a) $C_{max} = \dfrac{56.062 - 55.988}{2} = 0.037$ mm

$C_{min} = \dfrac{56.05 - 56}{2} = 0.025$ mm

b) $S = \frac{\mu N}{P}\left(\frac{r}{c}\right)^2 = \frac{120\ mPa\cdot s \cdot 18.33\frac{rev}{s} \cdot \frac{Pa}{1000\ mPa}}{1.53\ MPa \cdot 10^6\ Pa/MPa}\left(\frac{28}{0.025}\right)^2 = 1.8$

$\mu = 120\ mPa\cdot s$

$N = 1100\ \frac{rev}{min} \cdot \frac{min}{60\ s} = 18.33\ rev/s$

$P = \frac{W}{2rl} = \frac{2400\ N}{56\ mm \cdot 28\ mm} = 1.53\ MPa$

$r = 28\ mm = 0.028\ m$

c) $\frac{P}{P_{max}} = 0.5 \rightarrow P_{max} = 2P = 3.06\ MPa$

d) @ C_{max} $\quad S = \frac{120\ mPa\cdot s \cdot 18.33\frac{rev}{s} \cdot \frac{Pa}{1000\ mPa}}{1.53\ MPa \cdot 10^6\ Pa/MPa}\left(\frac{28}{0.037}\right)^2 = 0.82$

when S decreased to 0.82, $\frac{P}{P_{max}} \approx 0.45$

$P_{max} = \frac{1.53\ MPa}{0.45} = 3.4\ MPa$ → slightly higher pressure.

PROBLEM No. 5 (continued)