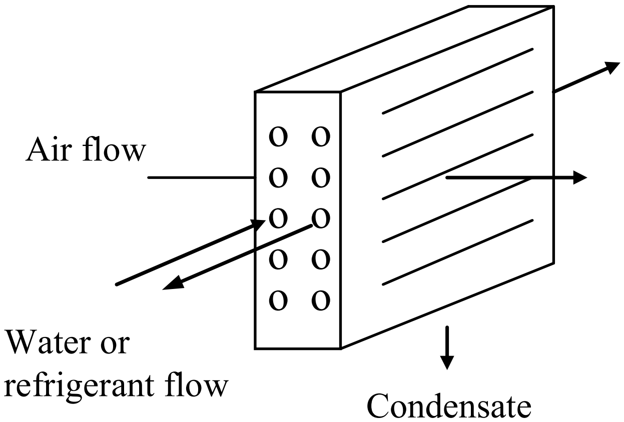
ME 418
Lecture 9 - Heat Exchanger Analysis & Design
In-Class Notes for Fall 2024

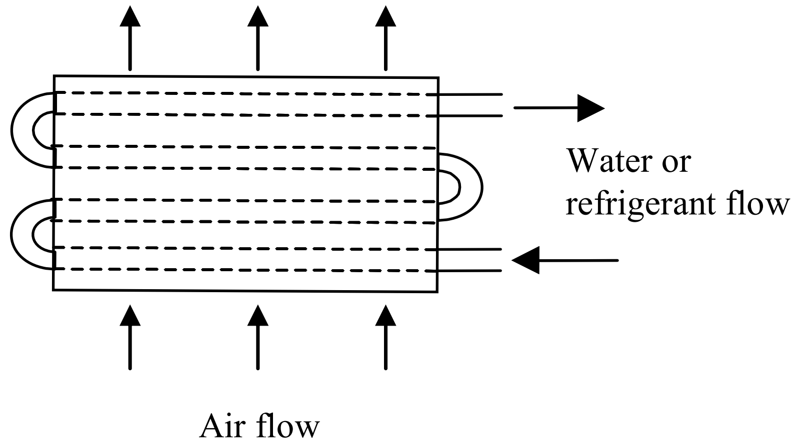
- Heat exchanger overview
- Overall heat exchanger conductance
- Heat transfer analysis – LMTD method
- Heat transfer analysis – effectiveness-NTU method
- Fin efficiencies
- Heat transfer coefficient
- Flow pressure drop

Heat Exchanger Types

Air-to-liquid

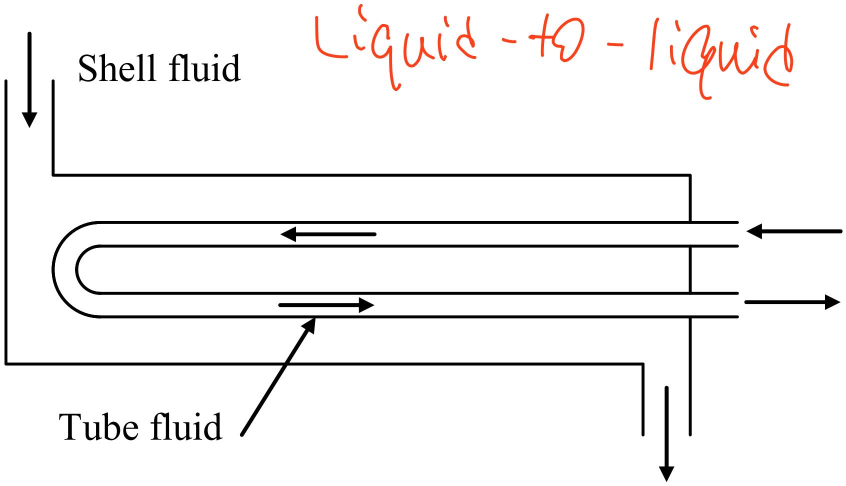
Coil used for heating or cooling air

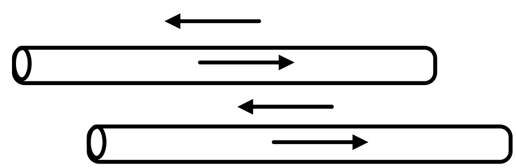




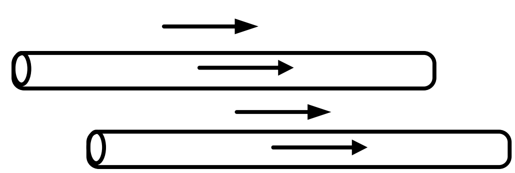
Shell and tube exchanger used for liquids

Liquid-to-liquid





Counter flow

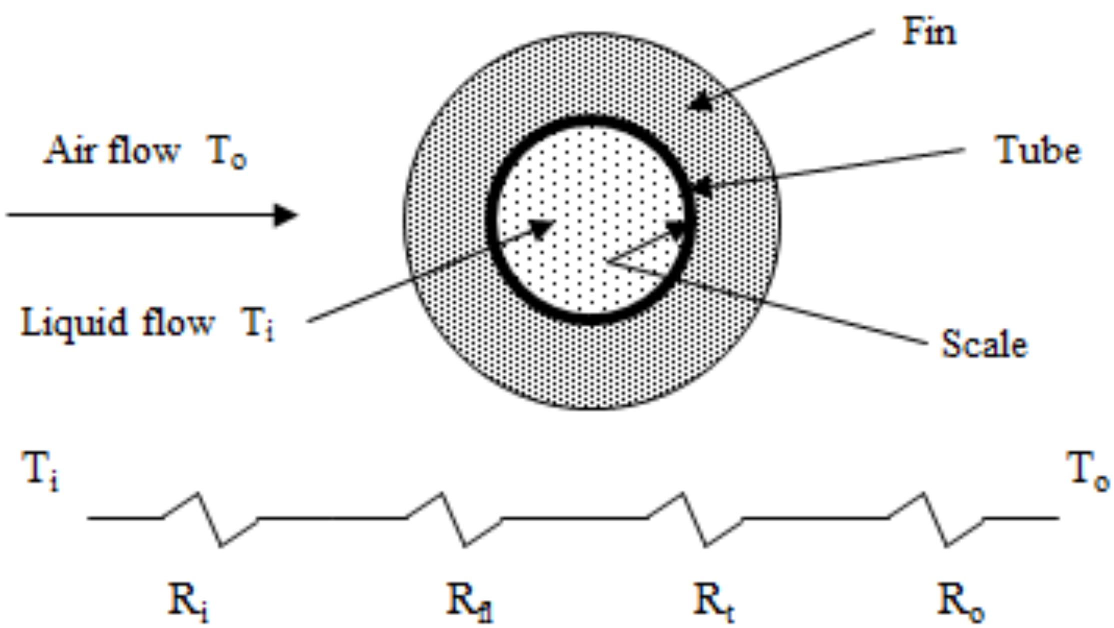


Parallel flow

Overall Heat Exchanger Conductance

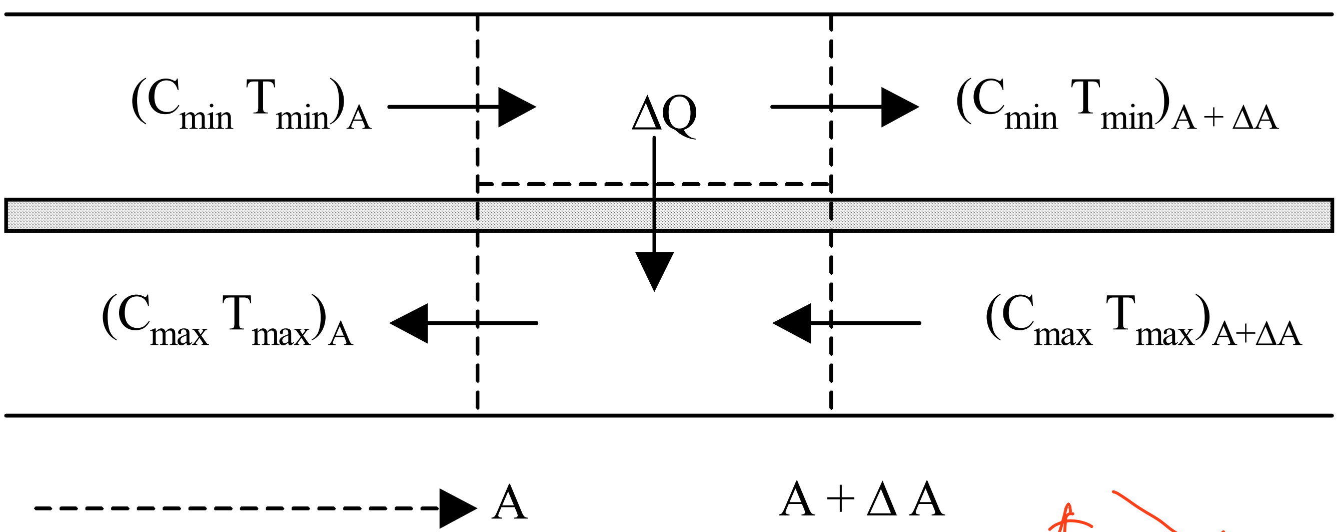
Total heat transfer resistance between two fluids in a finned tube heat exchanger

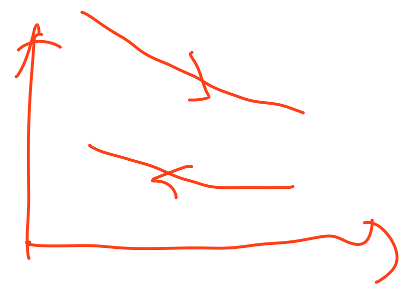
$$R_T = R_i + R_{fl} + R_t + R_o$$



- R_i: tube inside convection resistance
- R_{fl}: tube inside fouling resistance
- R_t: tube wall conduction resistance
- R_o: outside surface (fin+tube) convection resistance

Energy flows for a counter-flow heat exchanger





We define

$$C_{min} = \dot{m}_{min}\, c_{p,min} \quad \text{and} \quad C_{max} = \dot{m}_{max}\, c_{p,max}$$

Energy balance for the infinitesimal CV

steady-state

$$\left(C_{min}\ T_{min}\right)_A + \left(C_{max}\ T_{max}\right)_{A+\Delta A} - \left(C_{min}\ T_{min}\right)_{A+\Delta A} - \left(C_{max}\ T_{max}\right)_A = 0$$

$$\Updownarrow$$

$$C_{min}\frac{dT_{min}}{dA} = C_{max}\frac{dT_{max}}{dA}$$

Heat transfer between two small CVs

$$\Delta Q = U \Delta A \left(T_{min} - T_{max} \right)$$

Energy balance for one side

$$C_{min} \frac{dT_{min}}{dA} = -U \left(T_{min} - T_{max} \right)$$ → Tmin side

$$C_{max} \frac{dT_{max}}{dA} = -U \left(T_{min} - T_{max} \right)$$ → Tmax side

*Integrate above equations to find total heat transfer

Log-Mean-Temperature-Difference Method

Log-Mean-Temperature-Difference (LMTD) is defined as

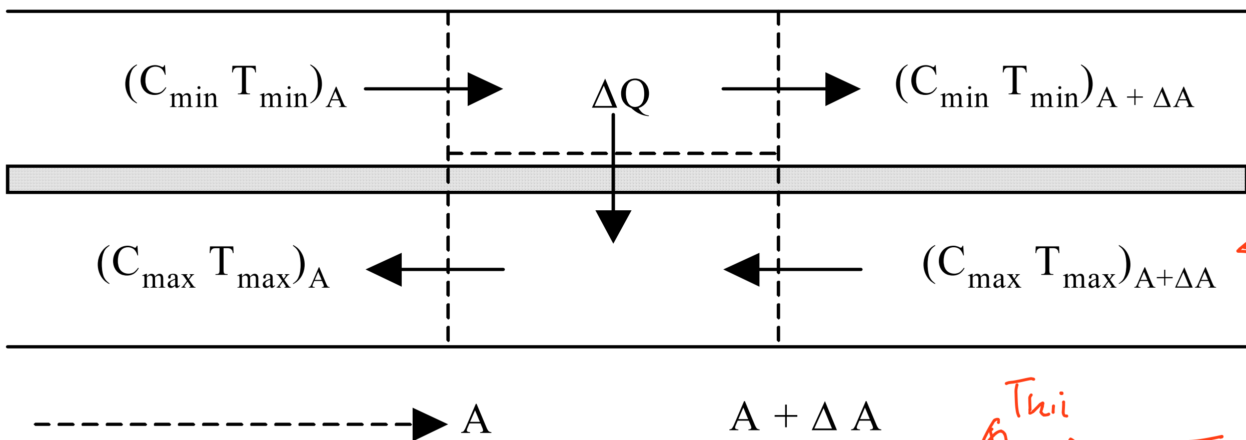
$$LMTD = \frac{\left(T_{h,in} - T_{c,out} \right) - \left(T_{h,out} - T_{c,in} \right)}{\ln \left(\dfrac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}} \right)}$$

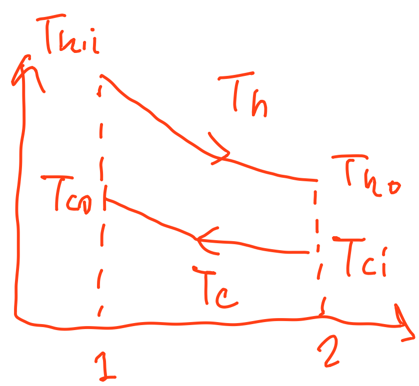
Heat transfer rate is calculated by

$$\dot{Q} = UA\,(LMTD)\,F$$

where F is a correction factor for flow types other than counterflow.

hot →





$$\Delta Q = U\Delta A\cdot(T_h - T_c)$$

$$= -C_h\cdot\Delta T_h = -C_c\cdot\Delta T_c$$

$C_h = m_h\cdot c_h$

negative in A-direction

$$\Rightarrow\ dT_h = -\frac{dQ}{C_h} \qquad dT_c = -\frac{dQ}{C_c}$$

$$\Rightarrow\ d(T_h - T_c) = dQ\cdot\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

$$= U\cdot dA\,(T_h - T_c)\cdot\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

$$\Rightarrow\ \frac{d(T_h - T_c)}{T_h - T_c} = U\,dA\cdot\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

$$(\ln x)' = \frac{1}{x}$$

Integrate

$$\Rightarrow\ \ln\left[\frac{(T_h - T_c)|_2}{(T_h - T_c)|_1}\right] = \ln\left(\frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}}\right) = U\cdot A\left(\frac{1}{C_c} - \frac{1}{C_h}\right) \quad (*)$$

$$= UA \cdot \frac{T_{co} - T_{ci} - T_{hi} + T_{ho}}{Q} \qquad \left\{ \begin{aligned} C_h \cdot (T_{hi} - T_{ho}) &= Q \\ &= C_c \cdot (T_{co} - T_{ci}) \end{aligned} \right.$$

$$\Rightarrow Q = UA \cdot \underbrace{\frac{T_{co} - T_{ci} - T_{hi} + T_{ho}}{\ln\left(\frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}}\right)}}_{\text{LMTD}} \qquad \Rightarrow \left\{ \begin{aligned} C_h &= \frac{Q}{T_{hi} - T_{ho}} \\ C_c &= \frac{Q}{T_{co} - T_{ci}} \end{aligned} \right.$$

<u>Effectiveness-NTU Method</u>

Overall energy balance

$$\dot{Q} = C_h \left(T_{h,i} - T_{h,o} \right) = C_c \left(T_{c,o} - T_{c,i} \right)$$

We define heat transfer effectiveness

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}}$$

where $\dot{Q}_{max} = C_{min} \left(T_{h,i} - T_{c,i} \right)$ is the max heat transfer.

Then the heat transfer rate is

$$\dot{Q} = \varepsilon\, C_{min}\left(T_{h,i} - T_{c,i}\right)$$

We further define capacitance rate ratio

$$C^* = \frac{C_{min}}{C_{max}} \quad \begin{matrix} = m_{min} \cdot C_{p\,min} \\ = m_{max} \cdot C_{p\,max} \end{matrix}$$

and number of transfer units (NTUs)

$$Ntu = \frac{UA}{C_{min}}$$

Then effectiveness can be calculated as

$$\varepsilon = f\left(C^*, Ntu\right)$$

Exp (*) $\Rightarrow$ $\dfrac{T_{ho} - T_{ci}}{T_{hi} - T_{co}} = \exp\left(UA \left(\frac{1}{C_c} - \frac{1}{C_h} \right) \right)$

$= \exp\left(\left(\underbrace{\frac{UA}{C_{min}}}_{NTU} \right) \left(\underbrace{\frac{C_{min}}{C_{max}}}_{C^*} - 1 \right) \right)$

$$= \exp\left(NTU \cdot (C^*-1)\right)$$

$$= \frac{T_{ho} - T_{hi} + T_{hi} - T_{ci}}{T_{hi} - T_{ci} + T_{ci} - T_{co}} \rightarrow \text{Driving potential}$$

Divide $T_{hi} - T_c$

$$= \frac{-\varepsilon + 1}{1 - C^* \cdot \varepsilon}$$

$$= \exp\left(NTU(C^*-1)\right)$$

$$\Leftrightarrow -\varepsilon + 1 = \exp(\cdot\cdot) - C^* \cdot \varepsilon \cdot \exp(\cdot\cdot)$$

$$\Leftrightarrow \varepsilon \cdot (C^* \exp(\cdot\cdot) - 1) = \exp(\cdot\cdot) - 1$$

$$\Leftrightarrow \varepsilon = \frac{\exp(NTU(C^*-1)) - 1}{C^* \cdot \exp(NTU(C^*-1)) - 1}$$

$$= f(NTU, C^*)$$

$$C_h(T_{ho} - T_{hi}) = C_c(T_{ci} - T_{co})$$

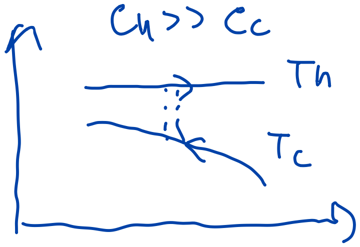
$$\Rightarrow T_{ho} - T_{hi} = \frac{1}{C^*}(T_{ci} - T_{co})$$

$$\Rightarrow \frac{T_{ho} - T_{hi}}{T_{hi} - T_{ci}} = -\varepsilon$$

$$= -\frac{1}{C^*} \cdot \frac{T_{ci} - T_{co}}{T_{hi} - T_{ci}}$$

Scenarios for C_h, C_c.

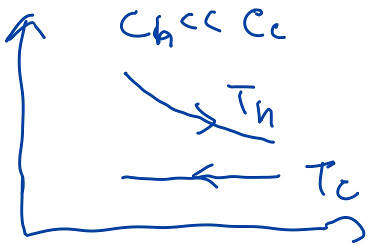
$C_h >> C_c$



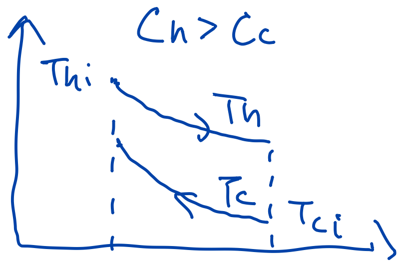
$C_h \Delta T_h = C_c \cdot \Delta T_c$

$C_h = \infty \rightarrow$ condensation

$C_h << C_c$



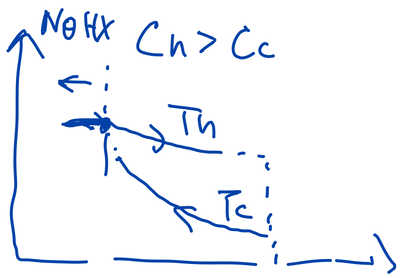
$C_c = \infty \rightarrow$ evaporation



$C_c \cdot \Delta T_c = C_h \cdot \Delta T_h$

↑ ↓

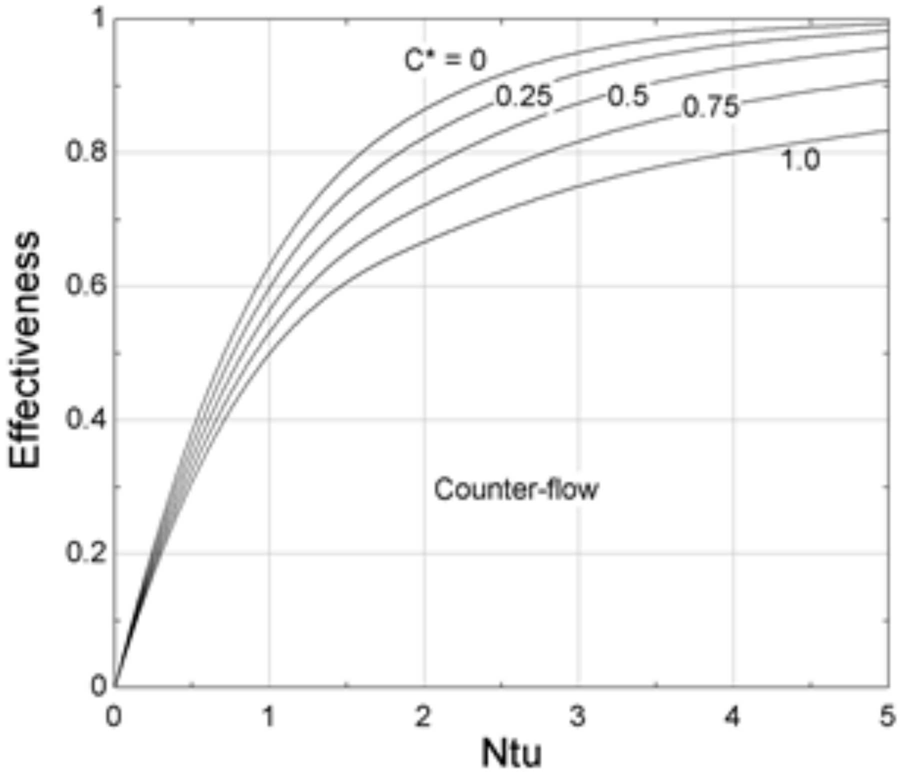
$T_{hi} - T_{ci}$ is driving potential



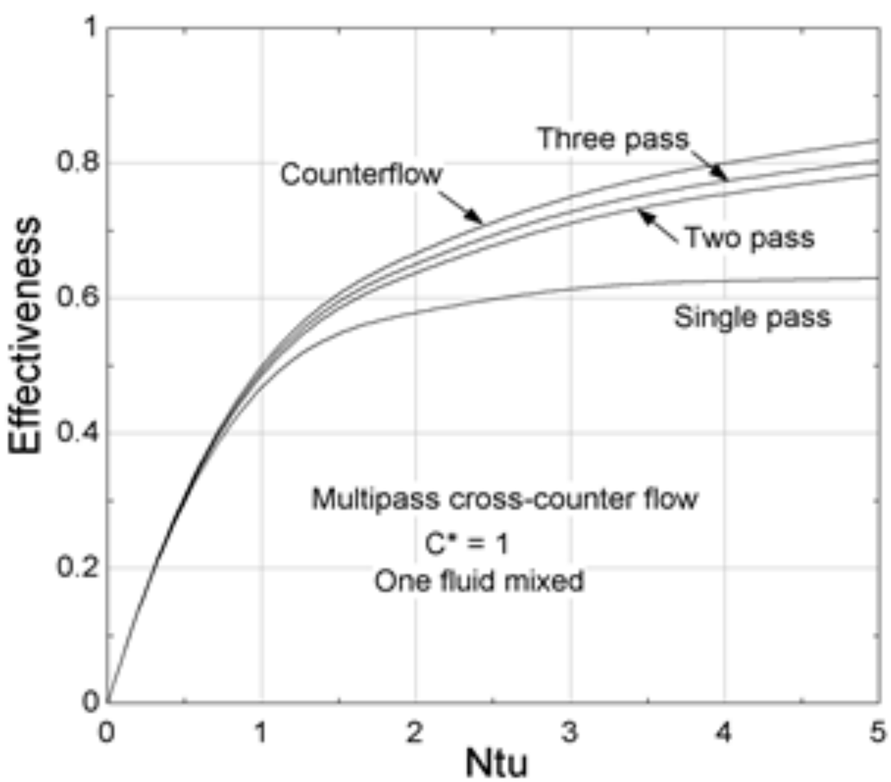
$T_{co} - T_{ci} = \boxed{T_{hi} - T_{ci}}$

ΔT_{max} is achieved on cold side or C_{min} side

Flow arrangement	Conditions	Effectiveness
All	$C^* = 0$	$\varepsilon = 1 - e^{-Ntu}$
Counterflow	$C^* \neq 1$	$\varepsilon = \dfrac{1 - e^{-Ntu(1-C^*)}}{1 - C^* e^{-Ntu(1-C^*)}}$
	$C^* = 1$	$\varepsilon = \dfrac{Ntu}{1 + Ntu}$
Crossflow, one fluid mixed	Single pass	$\varepsilon = \dfrac{1 - e^{-C^*(1-e^{-Ntu})}}{C^*}$
	Two pass	$\varepsilon = \left[1 - 0.0643\, C^* \left(1 - e^{-0.548\, Ntu}\right)\right] \left[\dfrac{1 - e^{-Ntu(1-C^*)}}{1 - C^* e^{-Ntu(1-C^*)}}\right]$
	Three pass	$\varepsilon = \left[1 - 0.0411\, C^* \left(1 - e^{-0.414\, Ntu}\right)\right] \left[\dfrac{1 - e^{-Ntu(1-C^*)}}{1 - C^* e^{-Ntu(1-C^*)}}\right]$
Shell and tube: even number of passes (2, 4, 6, ...) Crossflow		$\varepsilon = \dfrac{2}{(1 + C) + \sqrt{1 + C^{*2}} \left(\dfrac{1 + e^{-Ntu\sqrt{1+C^{*2}}}}{1 - e^{-Ntu\sqrt{1+C^{*2}}}}\right)}$
	Both fluids unmixed	$\varepsilon = 1 - \exp\left[\dfrac{Ntu^{0.22}}{C^*} \left(\exp\left(-C^* Ntu^{0.78}\right) - 1\right)\right]$



Counterflow



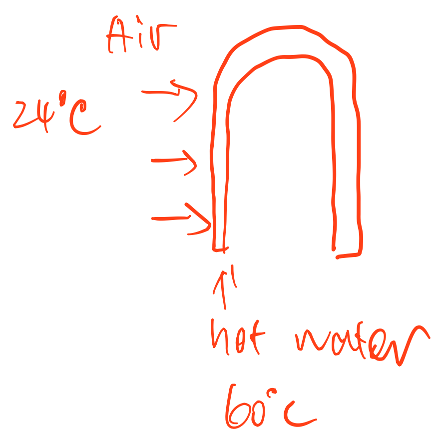
Multi-pass cross-flow with one fluid mixed

Notes:

- LMTD and ε-NTU methods are equivalent
- LMTD is easier to use when desired fluid outlet conditions are given
- ε-NTU is better when heat exchanger size and performance (i.e., UA) is given

Heat Exchanger Example: Determine the outlet temperatures and the heat transfer rate for a heating coil in which air is heated using hot water with two passes. The overall heat transfer conductance is 4 kW/C. The air stream enters at 24 C with a flow rate of 3 kg/s, and the water stream enters at 60 C with a flow rate of 1.0 kg/s.

uA



$$C_{air} = m_a \cdot C_{pa}$$

$$C_{wat} = m_w \cdot C_{pw}$$

$$C_{min} = \min(C_{air}, C_{wat})$$

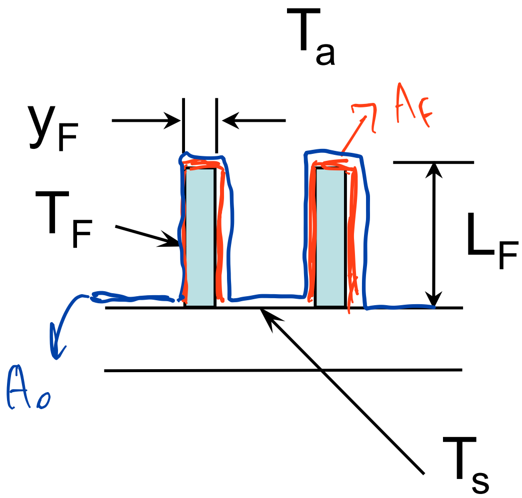
$$\Rightarrow \left.\begin{array}{l} C^* = \dfrac{C_{min}}{C_{max}} \\ NTU = \dfrac{UA}{C_{min}} \end{array}\right\} \Rightarrow \varepsilon$$

$$Q = \varepsilon \cdot C_{min} \cdot (T_{wi} - T_{ai})$$

$$T_{ao} = T_{ai} + \frac{Q}{C_{air}}$$

$$T_{wo} = T_{wi} - \frac{Q}{C_{wat}}$$

Fin Efficiencies



- fin heat transfer depends on local temperature difference $(T_F - T_a)$
- characterize finned surface heat transfer using a fin efficiency

Effective overall surface area

$$\overbrace{\underbrace{\eta_o}_{\textit{overall fin efficiency}} A_o}^{\text{effective area}} = \underbrace{(A_o - A_F)}_{\textit{exposed tube area}} + \underbrace{\eta_F}_{\textit{individual fin efficiency}} \overbrace{A_F}^{\textit{fin surface area}}$$

Individual Fin Efficiency

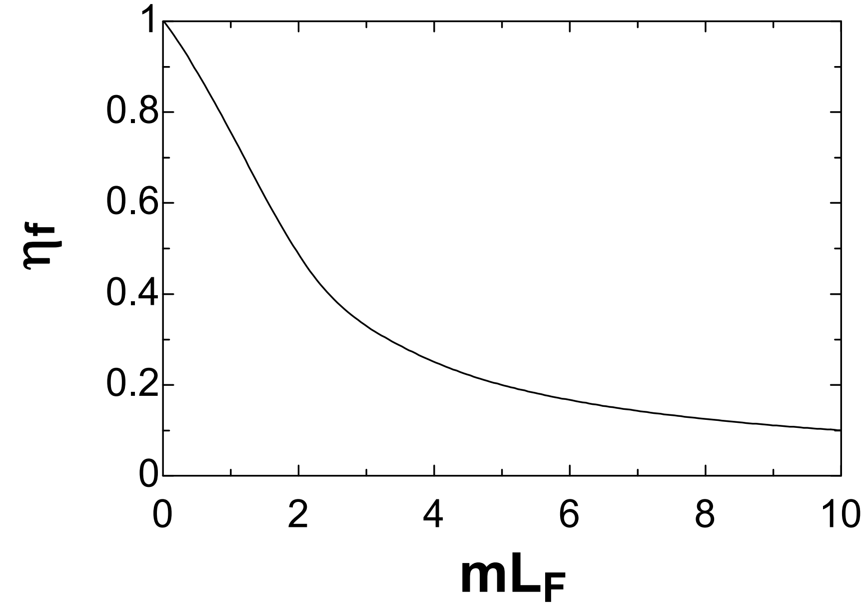
$$\eta_F = \frac{\text{heat transfer at base of fin}}{\text{maximum possible heat transfer if entire fin at } T_s}$$

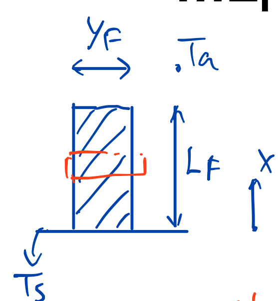
For straight fins,

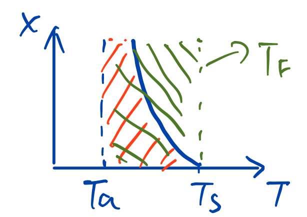
$$\eta_F = \frac{\tanh(mL_F)}{mL_F}$$

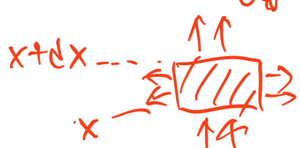
$$m = \left(\frac{2h_o}{k_F y_F}\right)^{1/2}$$

fin thermal conductivity









$Q_o = -k_F \cdot y_F \cdot \frac{dT_F}{dx}\Big|_{x+dx}$

$Q_c = 2h_o \cdot dx (T_F - T_a)$

$Q_i = -k_F \cdot y_F \cdot \frac{dT_F}{dx}\Big|_x$

$\eta_F = \frac{\int_0^{L_F} h_o \cdot dx \cdot (T_F - T_a)}{h_o (T_S - T_a) \cdot L_F} = \frac{\text{Red area}}{\text{green area}}$

Energy balance: $\Rightarrow$

$$2h_o \cdot dx (T_F - T_a) = k_F \cdot y_F \left(-\frac{dT_F}{dx}\Big|_x + \frac{dT_F}{dx}\Big|_{x+dx}\right)$$

Divide dx $\Rightarrow$

$$2h_o \cdot (T_F - T_a) = k_F \cdot y_F \cdot \frac{d^2T_F}{dx^2}$$

$$\frac{dT_a}{dx} = 0$$

$$= k_F \cdot y_F \cdot \frac{d^2(T_F - T_a)}{dx^2}$$

$$T_F = T_S \cdot \frac{\cosh(mL_F \cdot x)}{\cosh(mL_F)}.$$

Define $z = T_F - T_a \Rightarrow$

$$2h_o z = k_F \cdot y_F \cdot \ddot{z}$$

$$\Rightarrow \underbrace{\frac{2h_o}{k_F \cdot y_F}}_{m^2} \cdot z = \ddot{z} \rightarrow \text{2nd order ODE}$$

$$z_1 = e^{mx} \qquad z_2 = e^{-mx}$$

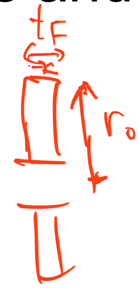
$$\Rightarrow m^2 \cdot z = \ddot{z} \qquad \Rightarrow z = c_1 \cdot e^{mx} + c_2 \cdot e^{-mx}$$

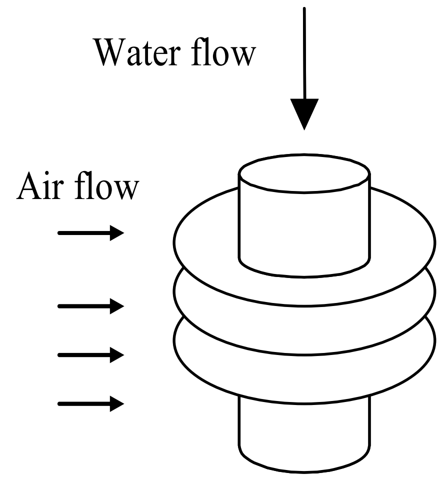
For circular fins,

$$\eta_f = \left[\frac{2 r_i}{m_f \left(r_{o,c}^2 - r_i^2 \right)} \right] \left[\frac{K_1(m_f r_i) I_1(m_f r_{o,c}) - I_1(m_f r_i) K_1(m_f r_{o,c})}{I_0(m_f r_i) K_1(m_f r_{o,c}) + K_0(m_f r_i) I_1(m_f r_{o,c})} \right]$$

where the effective fin radius and area are

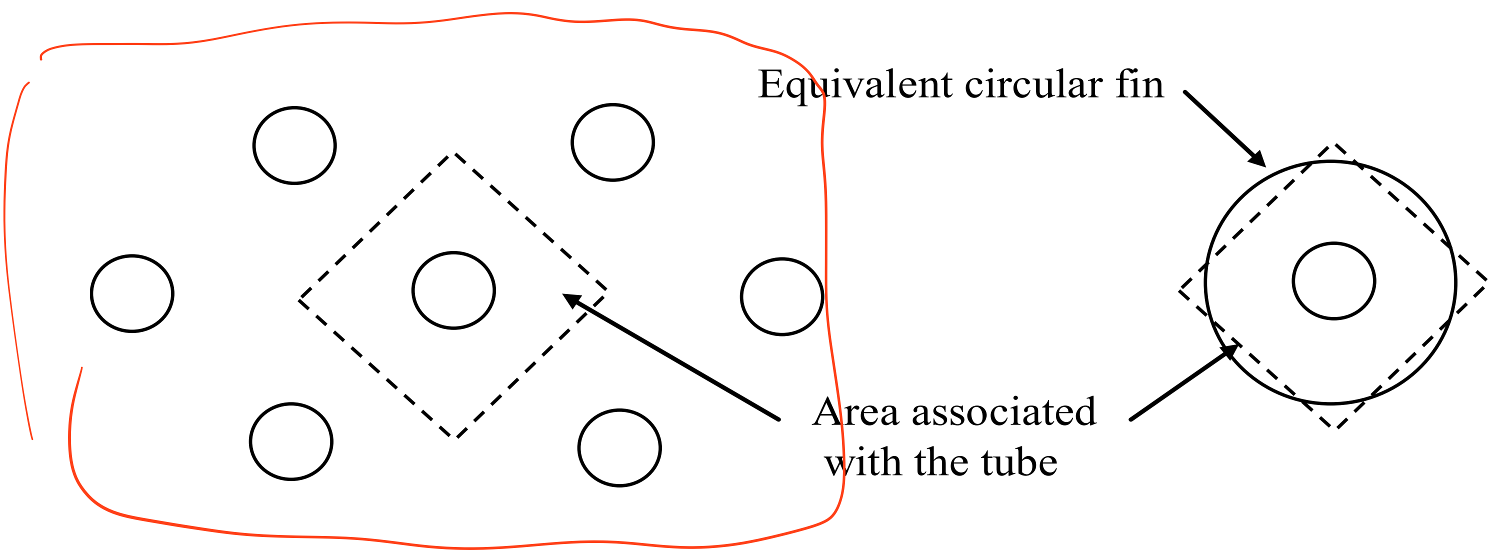
$$r_{o,c} = r_o + \frac{t_f}{2}$$





$$A_f = 2\pi \left(r_{o,c}^2 - r_i^2 \right)$$

Equivalent fin radius for plate-fin geometries



Fin Efficiency Example: Determine the fin efficiency, overall surface efficiency, and thermal resistance per foot of tube length for a cross-flow heat exchanger using finned tubes. The tube diameter is 0.774 inch. The fins are steel with a thickness of 0.012 inch, a diameter of 1.463 inch, and a pitch of 9.05 fins per inch. The conductivity of steel is 35 Btu/hr-ft-F. The heat transfer coefficient is 14. 4 Btu/hr-ft2-F."

d_i t_F, y_F k_F h_o

$$r_i = \frac{d_i}{2}$$

$$r_o = \frac{d_o}{2}$$

$$m_f = \sqrt{\frac{2h_o}{k_F \cdot y_F}}$$

$$r_{oc} = r_i + \frac{t_F}{2}$$

$$R = \frac{1}{\eta_o \cdot A_o \cdot h_o}$$

Tube Internal Single Phase Heat Transfer Coefficients and Pressure Drop

- Forced convection is generally involved in heat exchanger flow
- Correlations covered in Lectures 3 and 5 can be used for calculating heat transfer coefficient and pressure drop

Reynolds number is defined as

$$Re_{D_H} = \frac{\rho_f \ V \ D_H}{\mu_f} \quad \text{or} \quad Re_{D_H} = \frac{4 \ \dot{m}}{WP \ \mu_f}$$

where hydraulic diameter is

$$D_H = \frac{4 \ A_c}{WP}$$

Heat transfer coefficient is related to Nusselt number defined as

$$Nu_{D_H} = \frac{h_c D_H}{k_f}$$

where k_f is the fluid conductivity.

From Lecture 3, we can calculate **pressure drop** by

$$\Delta p = f \frac{L}{D_H} \frac{\rho_f V^2}{2} \quad \text{or} \quad \Delta p = f \frac{L}{D_H} \frac{G^2}{2\rho}$$

where G is mass velocity

$$G = \frac{\dot{m}}{A_c}$$

Heat transfer and friction factor relations for internal turbulent flow for Re > 2500

Smooth tubes	$Nu_{D_H} = 0.023\ Re_{D_H}^{0.8}\ Pr^{n}$ n = 0.4 heating or n = 0.3 cooling
Or (larger Re_D range)	$Nu_{D_H} = \dfrac{(f/2)\left(Re_{D_H} - 1000\right)Pr}{1 + 12.7(f/2)^{1/2}\left(Pr^{2/3} - 1\right)}$
Rough tubes	$Nu_{D_H} = \dfrac{\left(Re_{D_H}\ Pr(f/2)\right)}{1 + (f/2)^{1/2}\left(4.5\,Re_{D_H}^{0.2}\ Pr^{0.5} - 8.48\right)}$

Smooth tubes	$f = \dfrac{0.3164}{Re_{D_H}^{0.25}}$
Or (larger Re_D range)	$f = 0.0032 + \dfrac{0.221}{Re_{D_H}^{0.237}}$
Rough tubes	$\underline{Re_{D_H} < 10^6}$ $f^{-0.5} = 1.14 + 2\log\left(\dfrac{D_H}{\varepsilon}\right) - 2\log\left(1 + \dfrac{9.3}{Re\left(\dfrac{\varepsilon}{D_H}\right)f^{0.5}}\right)$ $\underline{Re_{D_H} > 10^6}$ $f^{-0.5} = 1.14 + 2\log\left(\dfrac{D_H}{\varepsilon}\right)$

Finned Surface Heat Transfer Coefficients and Pressure Drop

We define the ratio of free flow to frontal area

$$\sigma = \frac{A_c}{A_{fr}}$$

$A_c \rightarrow$ free flow area

$A_{fr} \rightarrow$ frontal area

Reynolds number $$N_R = Re_{D_H} = \frac{D_H G}{\mu}$$

$G = \frac{\dot{m}}{A_c}$ mass flux

The heat transfer coefficient is related to Sta~~n~~ton number

$$St = \frac{h_c}{Gc_p}$$

$\frac{\text{heat transfer}}{\text{thermal capacity}}$

Friction factor $$f = \frac{\rho\,\tau_0}{G^2/2}$$

Pressure drop can be calculated by

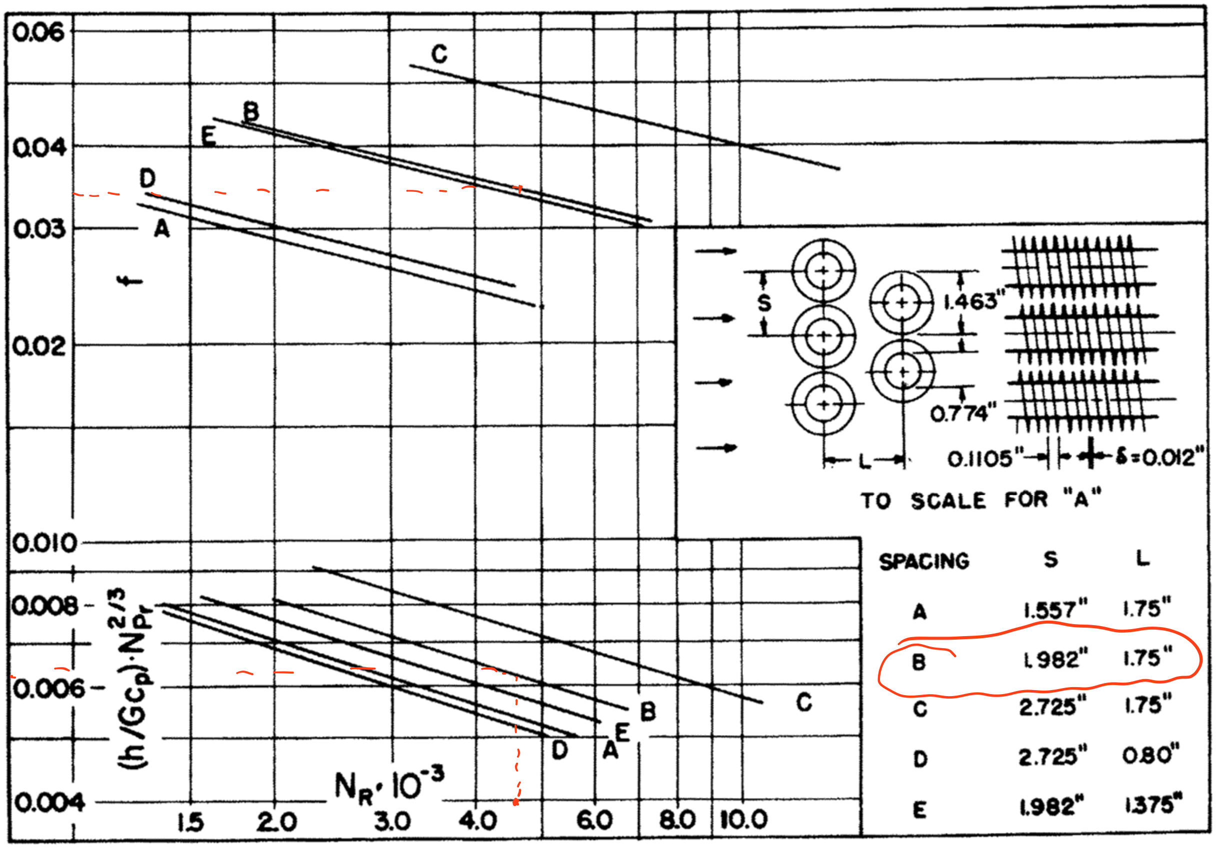
$$\Delta p = f\,\frac{A}{A_c}\,\frac{G^2}{2\rho}$$

$A \rightarrow$ heat transfer area

$A_c \rightarrow$ free-flow area

0.035

0.0063



Tube outside diameter = 0.774 in.

Fin pitch = 9.05 per in.

Fin thickness = 0.012 in.

Fin area/total area = 0.835 = $\frac{A_F}{A_o}$

	A	B	C	D	E
D_H hydraulic diameter: Flow passage hydraulic diameter, $4r_h$ =	0.01681	0.02685	0.0445	0.01587	0.02108 ft
Free-flow area/frontal area, σ = $\frac{A_c}{A_{fr}}$	0.455	0.572	0.688	0.537	0.572
$\frac{A}{V}$: Heat transfer area/total volume, α =	108	85.1	61.9	135	108 ft^2/ft^3

Note: Minimum free-flow area in all cases occurs in the spaces transverse to the flow, except for *D*, in which the minimum area is in the diagonals.

* Prandtl number raised to the two-thirds power is used to correlate the properties of other fluids.

Finned tube example: Determine the air-side convective heat transfer coefficient, thermal resistance, and pressure drop for a coil made of finned tubes with configuration B of figure above. The coil frontal area is 4 ft^2, there are four rows of coils, and the fins are made of aluminum. The airflow is 4000 cfm at a temperature of 75 F and 50 % relative humidity.

$D = \text{spacing} \cdot N_{row} = L \cdot N_{row}$ → depth of coil.

$V = A_{fr} \cdot D$

$\alpha = 0.85$

$V \cdot \alpha = A$ → total heat transfer area.

$D_H = 0.027$ ft

$A_{fr} \cdot \sigma = A_c$ → free flow area.

$$\frac{\dot{m}_a}{A_c} = G \qquad Re = \frac{D_H \cdot G}{\mu}$$

$$\frac{h_o}{G \cdot c_p} \cdot P_r^{2/3} = \text{Stanto} \cdot \text{Pr } \# \text{ from chart.}$$

$\eta_o = 0.78$

$$R_a = \frac{1}{\eta_o \cdot h_o \cdot A}$$

Find f from chart.

$$\Delta P = f \cdot \frac{A}{A_c} \cdot \frac{G^2}{2\rho}$$