ME 563-Fall 2024

Homework No. 2

Due: September 23, 2024 11:59 pm on Gradescope

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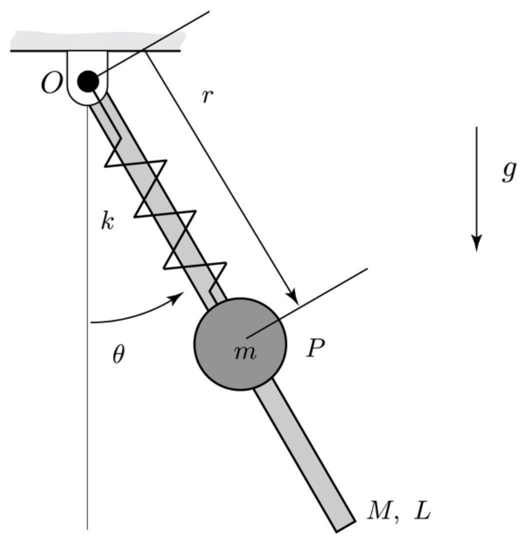
Homework Problem 2.1

A thin, homogenous bar having a mass of M and length of L is pinned to the ground at point O. A particle P of mass m is free to slide on the smooth surface of the bar. A spring of stiffness k and unstretched length of R_0 is attached at point O and particle P. Let r be the radial distance from O to P and Q be the rotation of the bar from a fixed vertical line.



a) Use Lagrange's equation to develop the EOM's for this two-DOF system using generalized coordinates of r and θ. Recall that the potential enrgy stored in a spring is related to the square of th stretch in the spring, where the stretch is equal to the difference between its actual length, and the unstretched length. Also, in writing down the velocity of vector of P, you may want to review the polar kinematic expressions for velocity.

b) Usin the equations of motion, determine the equilibrium values for r and θ.

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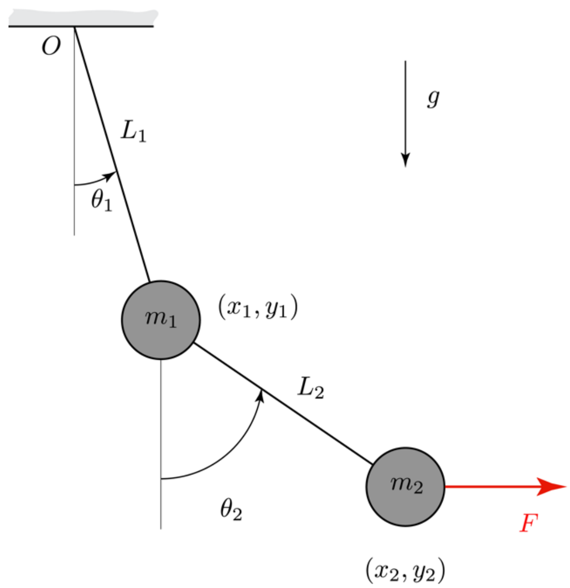
Homework Problem 2.2

A double pendulum consists of two bobs of mass m_1 and m_2, suspended by inextensible, massless strings of length L_1 and L_2.



a) Determine the expression for potential energy U and the generalized forces corresponding to F for the generalized coordinates. Use these results to determine the angles θ_1 and θ_2 corresponding to static equilibrium. Leave these angles in terms of the ratio F/mg.

b) Write down an expression for the kinetic energy T in terms of the generalized coordinates θ_1 and θ_2 and their time derivatives. From this expression, identify the elements m_{ij} , where:

$$T = \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} m_{ij}\dot{\theta}_i\dot{\theta}_j$$

c) Determine the mass matrix $[M]$ and the stiffness matrix $[K]$ corresponding small oscillations about the equilibrium state for $F/mg = 0$.

d) Determine the mass matrix $[M]$ and the stiffness matrix $[K]$ corresponding small oscillations about the equilibrium state for $F/mg = 2$. Compare these with those found in part c).

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Homework Problem 2.3

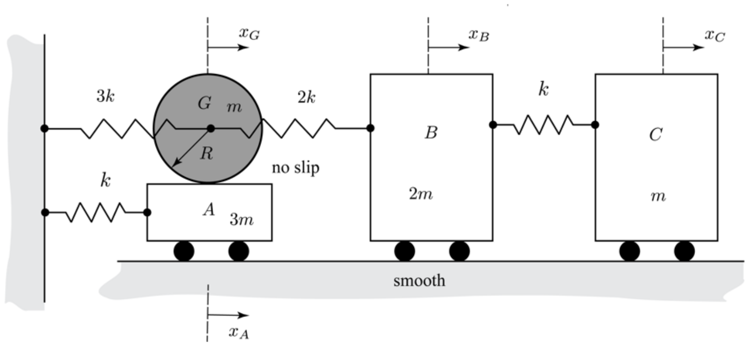
Consider the system below, whose motion is described by the absolute coordinates shown.

a) Write down the potential energy function U for this four-DOF system and use the following results from lecture to develop the stiffness matrix for the system:

$$K_{ij} = \left. \frac{\partial^2 U}{\partial q_i \partial q_j} \right|_{\mathbf{q}_0}$$

b) Use the method of influence coefficients to develop the flexibility matrix $[A] = [K]^{-1}$.

c) Check your results in a) and b) above by verifying that $[A][K]=[I]$ where $[I]$ is the identity matrix.



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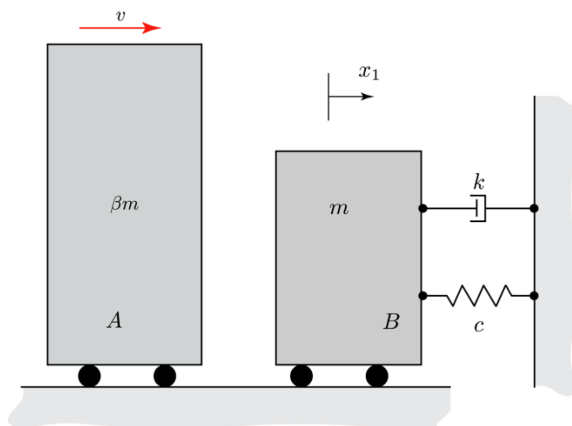
Homework Problem 2.4

Block A is initially moving to the right on a smooth, horizontal surface with a speed of v, with B at rest and the spring unstretched. Upon impact, at time $t = 0$, block A sticks to block B. For this problem use: $v = 10 \text{m/ sec}$, $m = 10 \text{ kg}$, $c = 60 \text{ kg / sec}$ and $k = 300 \text{ N / m}$.



a) Derive the differential equation of motion (EOM) for the system corresponding to $t > 0$.
b) Determine the numerical values for the undamped natural frequency and the damping ratio of the system corresponding to the EOM found in a) when β=1. How does the value change when β>1 and β<1.
c) Determine the response of the system for $t > 0$. HINT: You can use conservation of momentum to determine the speed of blocks A and B immediately after sticking when β=1. How does the value change when β>1 and β<1.
d) What is the maximum displacement of blocks A and B in the response found in c)? when β=1. How does the value change when β>1 and β<1.

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Homework Problem 2.5

a) Show that the logarithmic decrement is equal to

$$\delta = \frac{1}{n}\ln\frac{x_0}{x_n}$$

where x_n is the amplitude of vibration after n cycles have elapsed.

b) Show that by calculation that

$$A\sin(\omega_n t + \phi)$$

can be represented as

$$B\sin(\omega_n t) + C\cos(\omega_n t)$$

where B and C are functions of A and ϕ.

c) Solve

$$\ddot{x} - \dot{x} + x = 0$$

with initial conditions $x(0)=1$, and $v(0)=0$ for x(t) and sketch the time waveform.