Test 1

Name KEY

Pledge

I have neither given nor received aid on this examination.

Instructions:

• This is a closed-book, closed-notes exam.

• You are NOT allowed to use a programmable calculator during the exam.

• Please read the question on this exam carefully and answer only the questions I ask. Don't waste your time doing extra work, and don't skip any of the smaller questions within a problem. I can't grade what I can't see and I can't give partial credit for something in your head.

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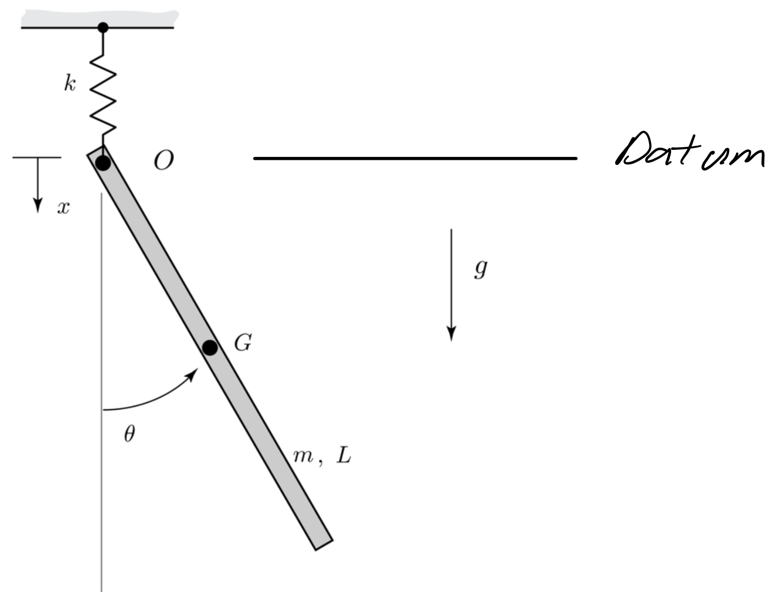
Test Problem 1 -30points

A bar is attached to a spring at pt O. The spring is constrained to deform purely in verical (x) direction.   The bar has mass m and mass moment of inerita about its center of gravtiy of $I^G = 1/12mL^2$. The coordinate x denotes the absolute position of the roller and θ the angular position of the bar.



a) Determine the expression for potential energy U in terms of the generalized coordinates x and θ and determine the equilibrium positions of the system.

b) Write down an expression for the kinetic energy T in terms of the generalized coordinates x and θ and their time derivatives. From this expression, identify the elements m_{ij}, where:

$$T = \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} m_{ij}\dot{q}_i\dot{q}_j$$

c) Determine the mass matrix $[M]$ and the stiffness matrix $[K]$ corresponding small oscillations about the equilibrium state.

a) $U = \frac{1}{2}kx^2 - mg(x + \frac{L}{2}\cos\theta)$

$\partial U/\partial x = kx - mg$, $\partial U/\partial\theta = mg\frac{L}{2}\sin\theta$

$\left.\frac{\partial U}{\partial q}\right|_{q_e} = \bar{0}$ $\begin{Bmatrix}\partial U/\partial x \\ \partial U/\partial\theta\end{Bmatrix} = \bar{0}$

1) $kx = mg$, $x_e = mg/k$

2) $mg\frac{L}{2}\sin\theta = 0$, $\theta_e = n\pi$

$n = 0, 1, 2, 3 \ldots$

$x_e = 0$, and $\theta_e = n\pi$, $n = 0, 1, 2, 3 \ldots \infty$

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b)

$$r_0 = x\hat{i} \quad , \quad \bar{r}_{G/O} = L/_2 \cos\theta\,\hat{i} + L/_2 \sin\theta\,\hat{j} \; ,$$

$$\vec{r}_G = \vec{r}_O + \vec{r}_{G/O} = (x + L/_2\cos\theta)\hat{i} + L/_2 \sin\theta\,\hat{j}$$

$$\vec{v}_G = (\dot{x} - \dot{\theta}L/_2 \sin\theta)\hat{i} + \dot{\theta}L/_2 \cos\theta\,\hat{j}$$

$$T = 1/_2\, m\,\vec{v}_G \cdot \vec{v}_G + 1/_2 I_G \dot{\theta}^2 = 1/_2\, m\,(\dot{x}^2 - \dot{x}\dot{\theta}L\sin\theta + \dot{\theta}L^2/_4) + 1/_{12} mL^2\dot{\theta}^2$$

$m_{11} = m$,

$m_{12} = m_{21} = -\frac{mL}{2}\sin\theta$, and

$m_{22} = mL^2/_4$

c)
$$[m] = m\begin{bmatrix} 1 & -L/_2 \cancelto{0}{\sin(n\pi)} \\ -L/_2 \cancelto{0}{\sin(n\pi)} & L^2/_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & L^2/_3 \end{bmatrix}$$

$$k_{11} = \left.\frac{\partial^2 U}{\partial x^2}\right|_{\vec{q}_e} = k \;, \quad k_{12} = \left.\frac{\partial U}{\partial x \partial \theta}\right|_{\vec{q}_e} = k_{21} = 0$$

$$k_{22} = \left.\frac{\partial^2 U}{\partial \theta^2}\right|_{\vec{q}_e} = mgL/_2 \cos(n\pi) = mg\,L/_2(-1)^n$$

$$k = \begin{bmatrix} k & 0 \\ 0 & mgL/_2(-1)^n \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & (-1)^n mgL/_2 \end{bmatrix}$$

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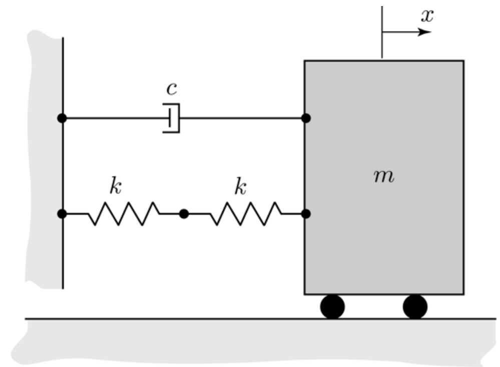
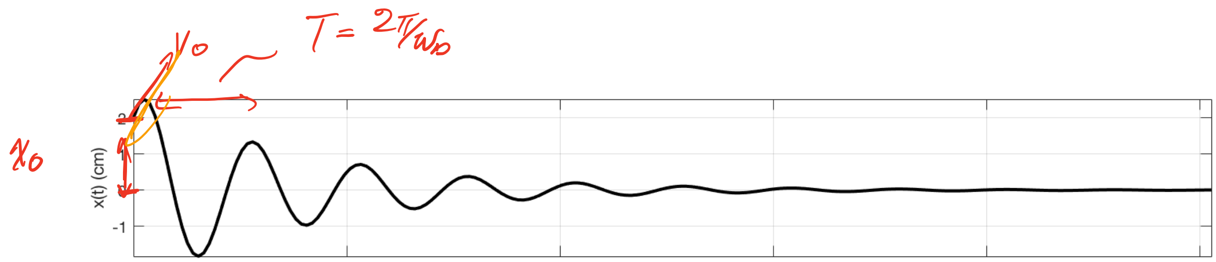
Test Problem 2 -20 points

A single degree of freedom systems has system parameters of m =100 kg, k= $25\sqrt{2}$ N/m, and c =50 kg/s, The system has initial condition x_0= 2cm, and v_0=4cm/s.

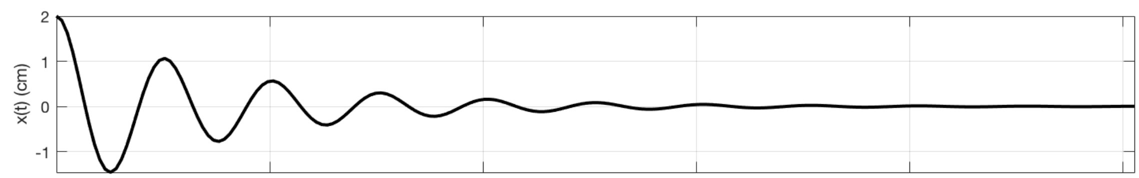
a) Determine the natural frequency of the system.
b) Determine the damping ratio of the system.
c) Determine the damped natural frequency of the system.
d) Indicate the correct plot of the response of system. On this label the initial conditions and the period of oscillations of the system, and the length of the period.



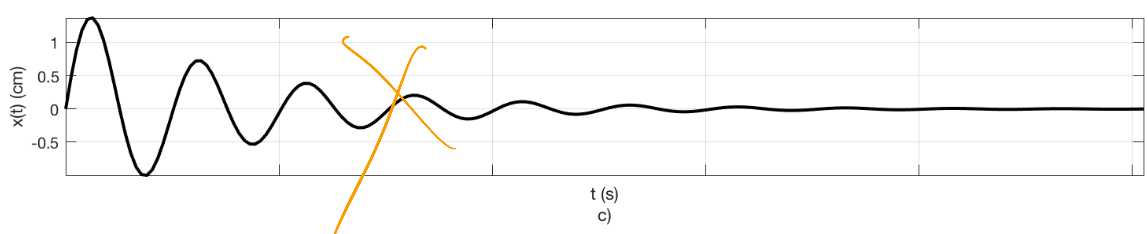
$T = 2\pi/\omega_D$



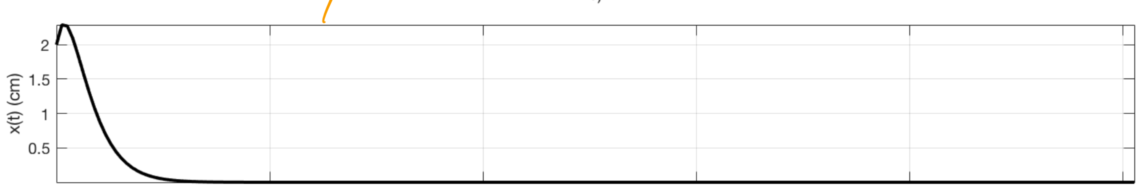
a)



b)



c)



d)

a) $1/k_{eq} = 1/k + 1/k \longrightarrow k_e = k^2/k2 = k/2 = \frac{25\sqrt{2}}{2}$ N/m $= 17{,}67$ N/m

$w_n = \sqrt{k_e/m} = \sqrt{25\sqrt{2}/2\cdot 100} = \sqrt{\sqrt{2}/8} = 0{,}4204$ rad/s

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b) $2\zeta\omega_n = c/m$

$$\zeta = c/2m\omega_n = c/2m\sqrt{K_{eq}/m} = c/2\sqrt{mK_{eq}}$$

$$= c/2\sqrt{mK/2} = \frac{50}{2\sqrt{100(25\sqrt{2}/2)}} = \underline{0.5946}$$

underdamped

c) $\omega_D = \omega_n\sqrt{1-\zeta^2} = 0.424\sqrt{1-0.5946^2} = \underline{0.3380 \text{ rad/s}}$

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Test Problem 3-20 points

A 2-DOF system has the follow equations of motion.

$$m\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\ \ddot{x}_2\end{Bmatrix}+k\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}x_1\\ x_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

Determine the natural frequencies and mode shapes of the system.

Assume $\bar{x} = \bar{X} e^{\lambda t} = \bar{X} e^{i\omega t}$

$[m]\ddot{\bar{x}} + [k]\bar{x} = \bar{0} \rightarrow \left(-\omega^2[m] + [k]\right)\bar{X}\cancel{e^{i\omega t}} = 0$

Natural Frequencies

$$\begin{bmatrix}-\omega^2 m + k & -k\\ -k & -\omega^2 m + k\end{bmatrix}\begin{Bmatrix}X_1\\ X_2\end{Bmatrix} = \bar{0}$$

$(-\omega^2 m + k)^2 - (k)^2 = (\omega^4 m^2 - \omega^2(2mk) + k^2 - k^2) = 0$

$\omega^4 - 2mk\,\omega^2 = \omega^2(\omega^2 m^2 - 2mk) = 0$

$\omega^2 = 0 \qquad \omega^2 m^2 - 2mk = 0$

$\underline{\omega = 0}$ and $\underline{\omega = \sqrt{2}\sqrt{k/m}}$ ~ Rigid body mode $\omega = 0$

Modal Shapes

$(-\omega^2 m + k)X_1 - kX_2 = 0 \rightarrow \dfrac{X_2}{X_1} = \dfrac{\omega^2 m + k}{k} = -\omega^2 m/k + 1$

$\bar{X} = \begin{Bmatrix}1\\ \omega^2 m/k + 1\end{Bmatrix}$

$\omega = 0,\ \bar{X}^1 = \begin{Bmatrix}1\\ 1\end{Bmatrix},\ \omega = \sqrt{2}\sqrt{k/m},\ \bar{X}^2 = \begin{Bmatrix}1\\ -1\end{Bmatrix}$