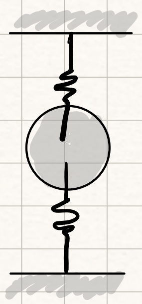
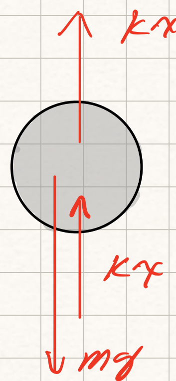
Problem #1

System (a)





$$+\downarrow \; \Sigma F_x: \; -kx - kx + mg = m\ddot{x}$$

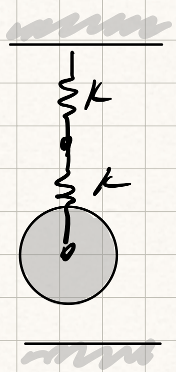
$$m\ddot{x} + 2kx = +mg$$

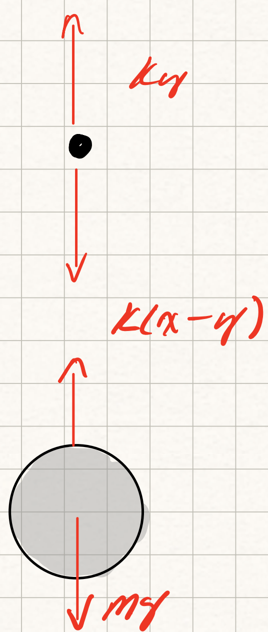
$$k_{eq} = 2k \quad \text{in parallel}$$

at equilibrium $\ddot{x} = \dot{x} = 0, \; x = x_s$

$$2kx_{ST} = mg \longrightarrow x_{ST} = \frac{mg}{2k} \qquad T$$

System (b)





$$+\downarrow \; \Sigma F_y: \; -ky + k(x-y) = 0$$

$$\downarrow \; \Sigma F_x: \; -k(x-y) - mg = m\ddot{x}$$

$$-ky + kx - ky = 0$$

$$2ky = kx$$

$$y = \tfrac{1}{2}x$$

$$-k(x - \tfrac{1}{2}x) - mg = m\ddot{x}$$

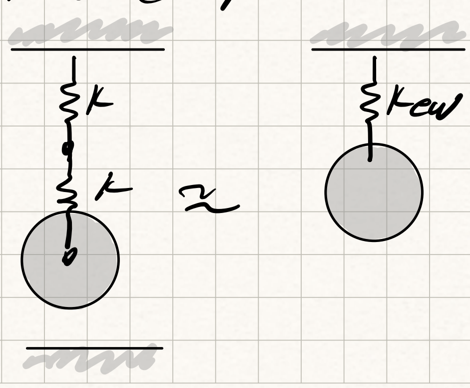
$$-\tfrac{k}{2}(x) - mg = m\ddot{x}$$

$$m\ddot{x} + \tfrac{k}{2}x = mg \; , \qquad k_{eq} = \tfrac{k}{2} \quad \text{in series}$$

at equilibrium $\ddot{x} = \dot{x} = 0, \quad x = x_{ST}$

$$\tfrac{k}{2}x_{ST} = mg \longrightarrow x_{ST} = 2mg/k$$

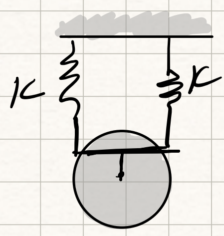
or directly realize

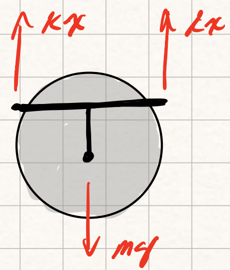


where : $\dfrac{1}{k_{eq}} = \dfrac{1}{k} + \dfrac{1}{k}$ series

$$k_{eq} = \frac{kk}{k+k} = \frac{k}{2}$$

System (c)





$+\downarrow \Sigma F_x: \; -kx - kx + mg = m\ddot{x}$

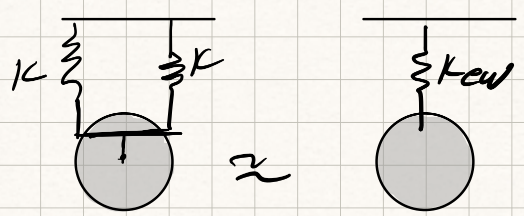
$m\ddot{x} + 2kx = +mg$

$k_{eq} = 2k$ in parallel

at equilibrium $\ddot{x} = \dot{x} = 0$

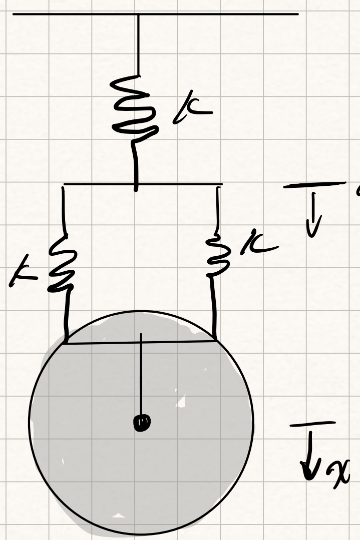
$2k x_{st} = mg \longrightarrow x_{st} = mg/2k$

or directly realize

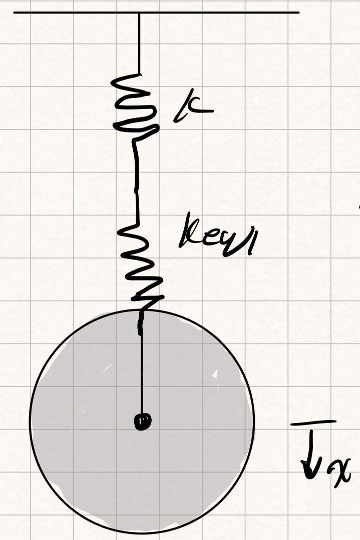


$k_{eq} = k + k = 2k$ parallel

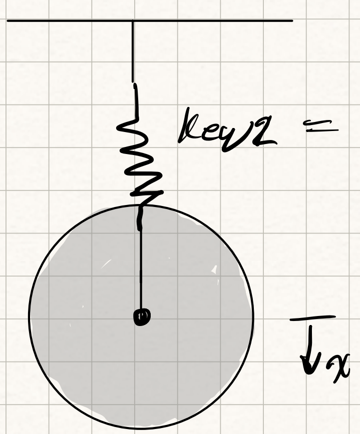
System (d)



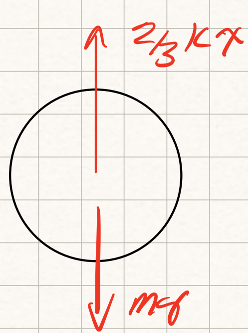
$\approx$



$k_{eq_1} = k + k = 2k$



$$k_{eq2} = \frac{k\,k_{eq1}}{k + k_{eq1}} = \frac{2k^2}{k+3k} = \tfrac{2}{3}k$$



$+\downarrow \Sigma F_{xi} \quad -\tfrac{2}{3}kx + mg = m\ddot{x}$

$m\ddot{x} + \tfrac{2}{3}kx = mg$

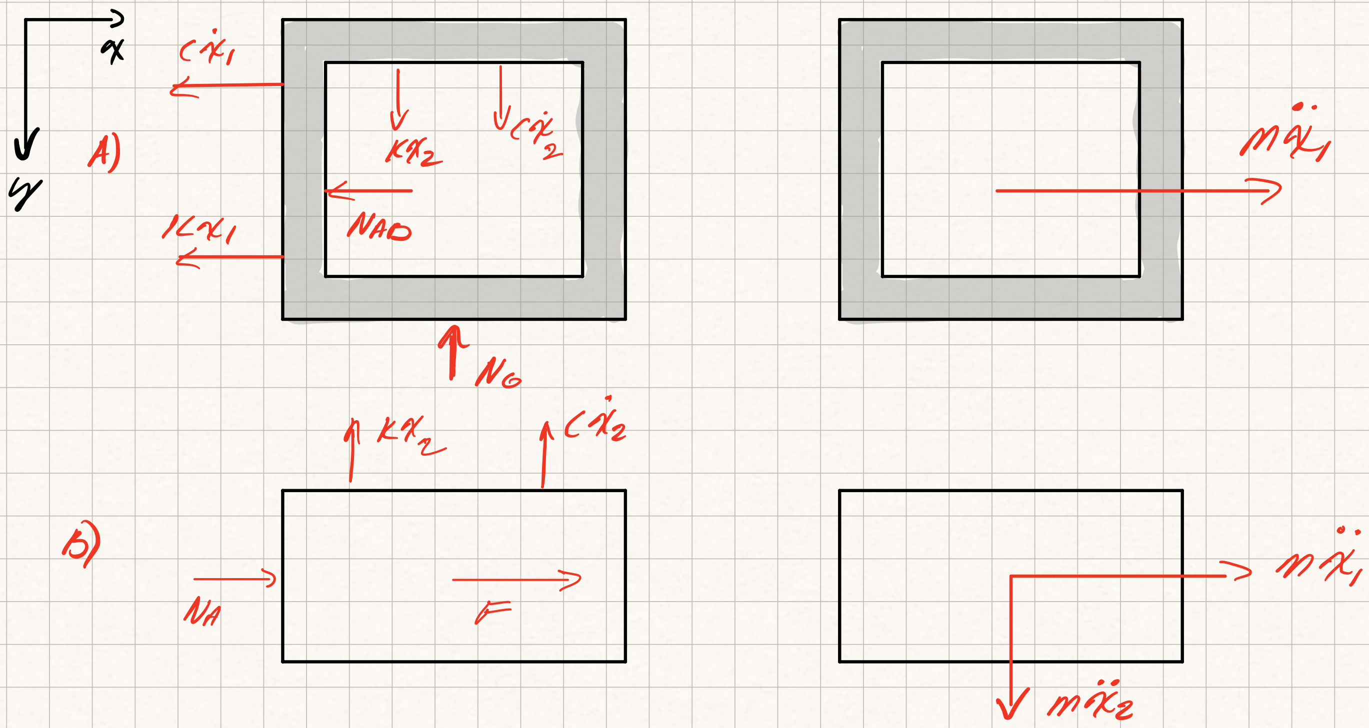
at equilibrium $\ddot{x} = \dot{x} = 0$

$\tfrac{2}{3}k x_{ST} = mg$

$x_{ST} = \tfrac{3}{2}\, mg/k$

System (b) deflects the most.

Problem #2



Block A: $\xrightarrow{+} \Sigma F_x: \; -c\dot{x}_1 - kx_1 - N_{AB} = m\ddot{x}_1$ 1)

Block B: $\xrightarrow{+} \Sigma F_x: \; N_{AB} + F = m\ddot{x}_1$ 2)

$+\downarrow \; \Sigma F_y: \; -c\dot{x}_2 - kx_2 = m\ddot{x}_2$ 3)

2) $N_{AB} = m\ddot{x}_1 - F$

1) $-c\dot{x}_1 - kx_1 - m\ddot{x}_1 + F = m\ddot{x}_1$

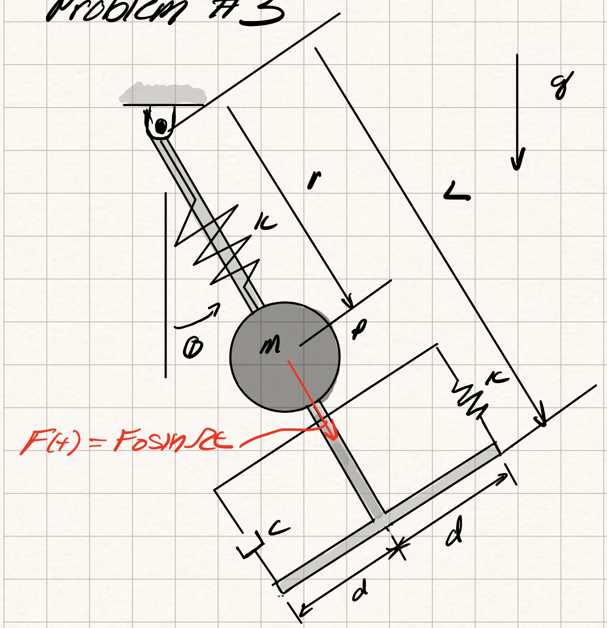
↳ $2m\ddot{x}_1 + c\dot{x}_1 + kx_1 = F$

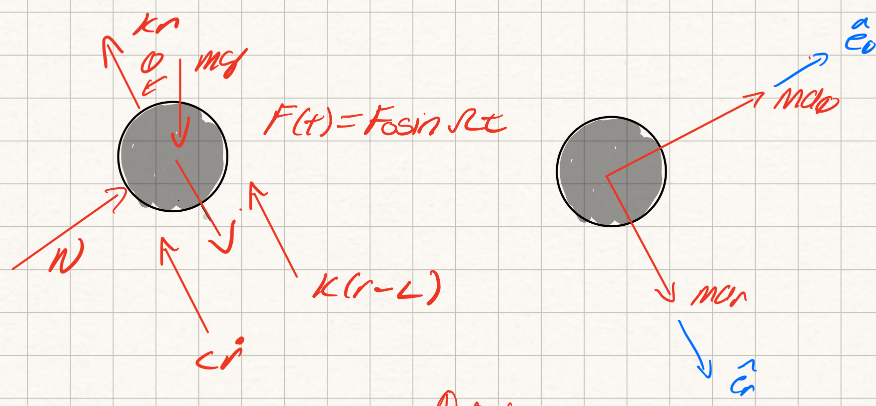
$m\ddot{x}_2 + c\dot{x}_2 + kx_2 = 0$

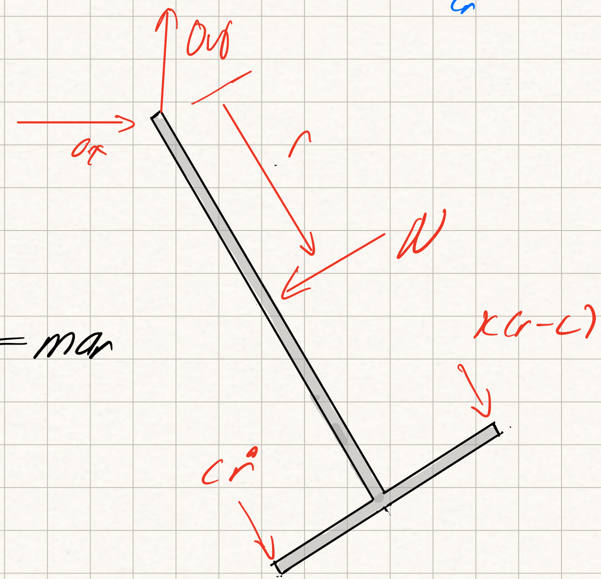
In matrix form

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

Problem #3







$$+\searrow \ \Sigma F_R: \ -kr - c\dot{r} - k(r-L) + F(t) + mg\cos\theta = m a_r$$

$$+\nearrow \ \Sigma F_\theta: \ -mg\sin\theta + N = m a_\theta$$

$$\curvearrowleft + \ \Sigma M_O: \ +Nr - c\dot{r}d + k(r-L)d = 0$$

$$N = \frac{c\dot{r}d - k(r-L)d}{r}$$

$$-2kr - c\dot{r} + kL + F(t) + mg\cos\theta = m a_r$$

$$-mg\sin\theta + \frac{c\dot{r}d}{r} - \frac{k(r-L)d}{r} = m a_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

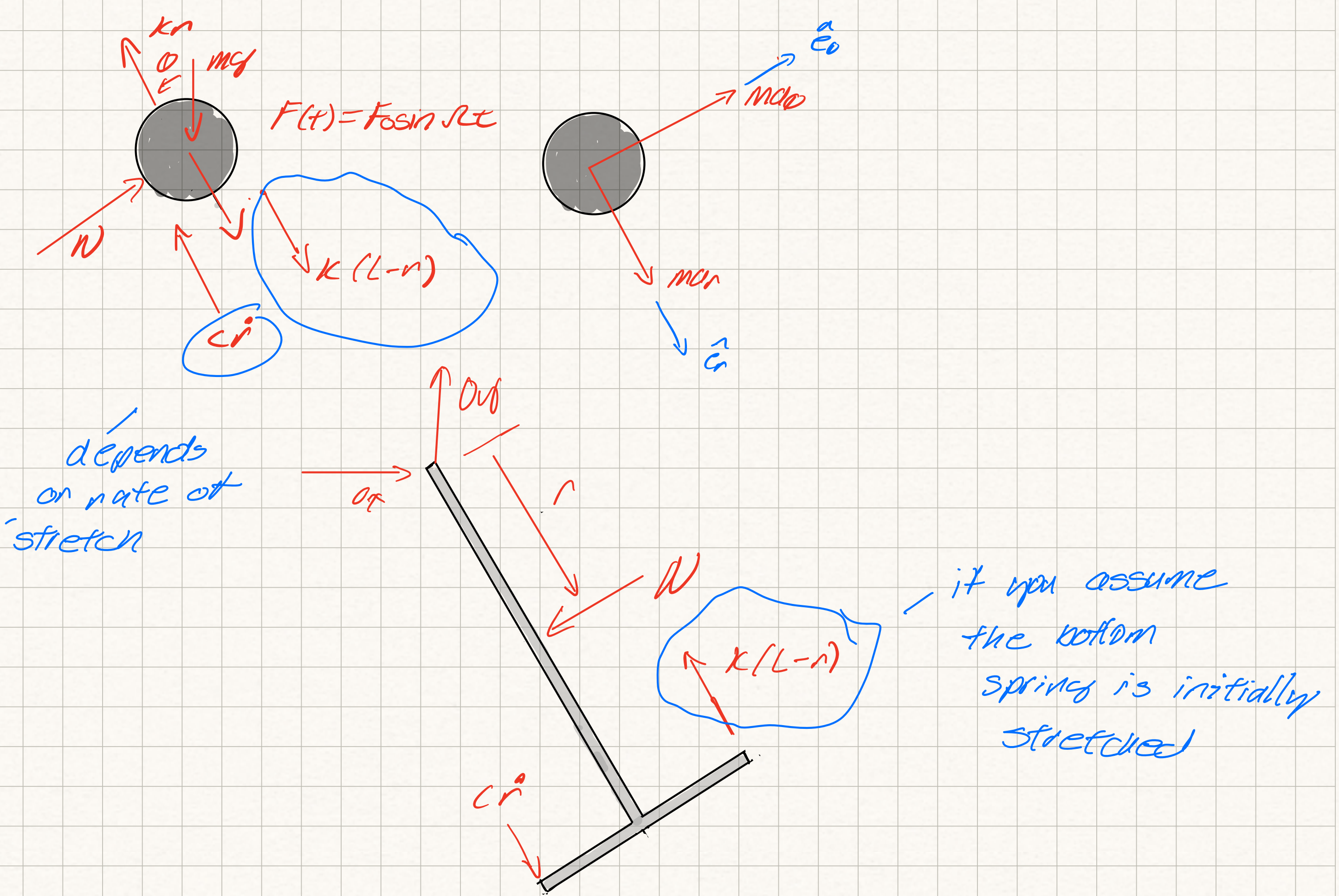
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$m\ddot{r} - mr\dot{\theta}^2 + c\dot{r} + 2kr + mg\cos\theta = kL + F_0\sin\Omega t$$

$$mr\ddot{\theta} + 2m\dot{r}\dot{\theta} + \frac{c\dot{r}d}{r} + \frac{kLd}{r} + mg\sin\theta = -kd$$

~ Ans

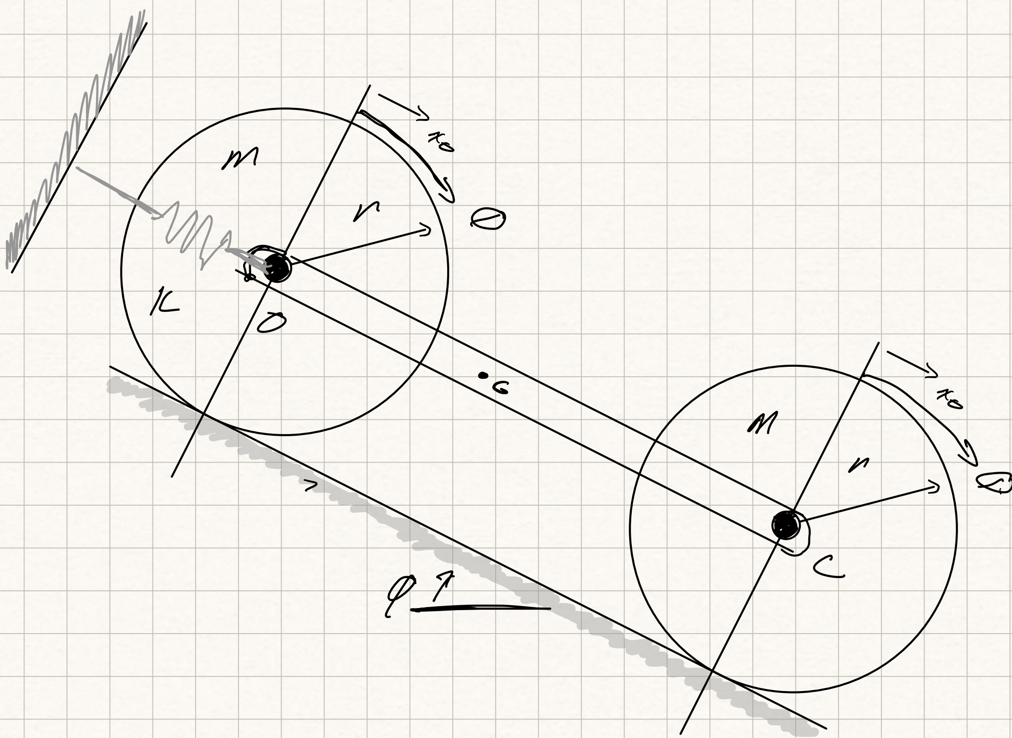
Note the wording was confusing, I will accept an alternate FBD

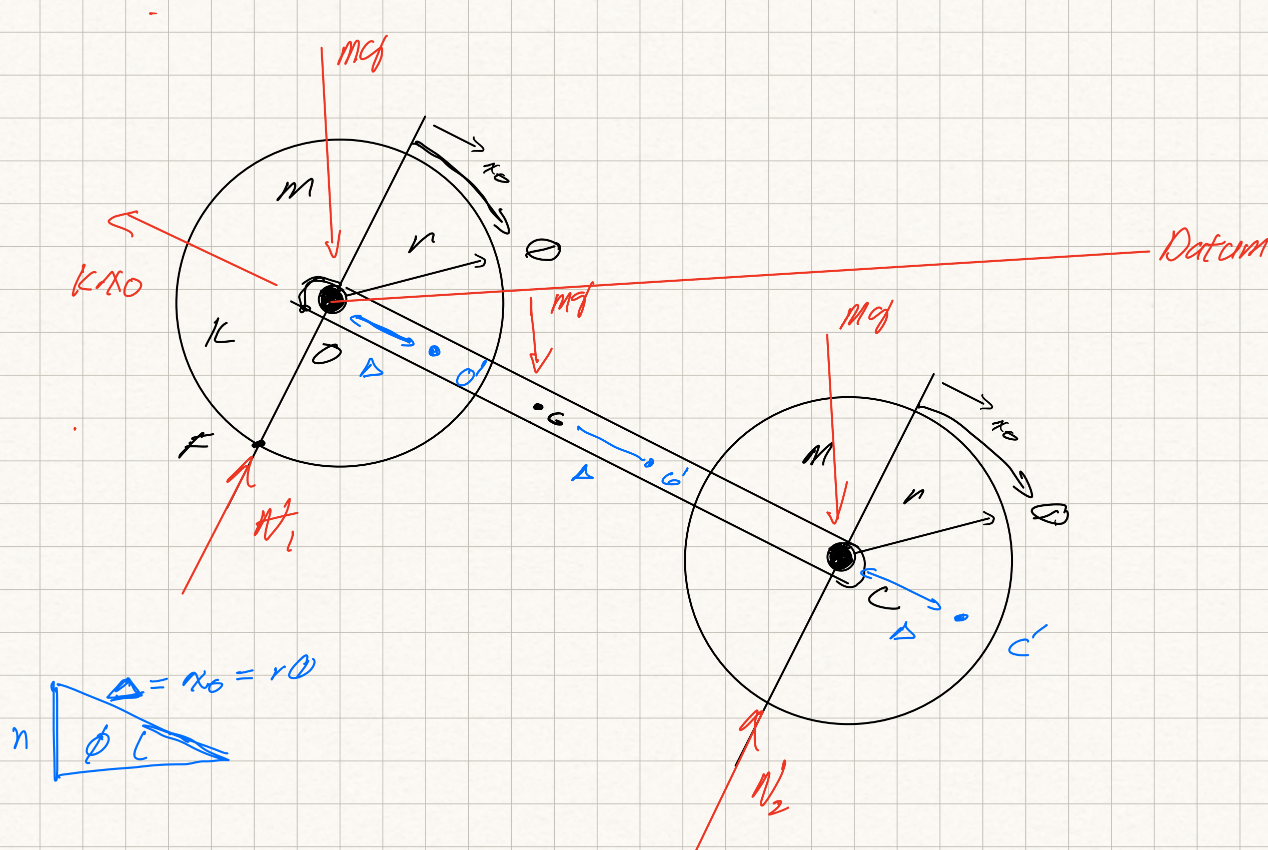


Will give the same answer

Problem 4

a)





$\Delta = x_0 = r\theta$

$$T = \frac{1}{2} m \vec{v}_O \cdot \vec{v}_O + \frac{1}{2} m \vec{v}_C \cdot \vec{v}_C + \frac{1}{2} I_O \dot{\theta}^2 + \frac{1}{2} I_G \dot{\theta}^2 + \frac{1}{2} m \vec{v}_G \cdot \vec{v}_G$$

$$I_O = I_C = I_G = \frac{1}{2} m r^2$$

$$\vec{V}_C = \vec{V}_F + \vec{\omega}_{Disk} \times \vec{r}_{C/F} = \vec{V}_O$$

$$\vec{V}_c = \vec{0} + \dot{\theta}\hat{k} \times r\hat{j} = -r\dot{\theta}\hat{i}$$

Furthermore $\vec{V}_C = \vec{V}_O = \vec{V}_G$

$$T = \tfrac{1}{2} m r^2 \dot{\theta}^2 + \tfrac{1}{2} m r^2 \dot{\theta}^2 + \tfrac{1}{2}(\tfrac{1}{2} m r^2)\dot{\theta}^2$$
$$+ \tfrac{1}{2}(\tfrac{1}{2} m r^2)\dot{\theta}^2 + \tfrac{1}{2} m r^2 \dot{\theta}^2 =$$

$$\tfrac{1}{2}(4mr^2)\dot{\theta}^2$$

$$U = -mgh - mgh - mgh + \tfrac{1}{2}kx_0^2 = -3mgh = -3mg\Delta\sin\phi$$
$$= -3mg(r\theta)\sin\phi + \tfrac{1}{2}kr^2\theta^2$$

$$T + U = \tfrac{1}{2}(4mr^2)\dot{\theta}^2 - 3mg\sin\phi(r\theta) + \tfrac{1}{2}kr^2\theta^2$$

$$\frac{dT}{dt} + \frac{dU}{dt} = 4mr^2\dot{\theta}\ddot{\theta} - 3mg\sin\phi\, r\dot{\theta} + kr^2\theta\dot{\theta} = 0$$

$$(4mr^2\ddot{\theta} + kr^2\theta)\dot{\theta} = 3mgr\sin\phi\,\dot{\theta}$$

$$\ddot{\theta} + \frac{k}{4m}\theta = \frac{3}{4}\frac{g}{r}\sin\phi$$

Ans