ME 563-Fall 2024

Homework No. 3

Due: October 23, 2024, 11:59 pm on Gradescope

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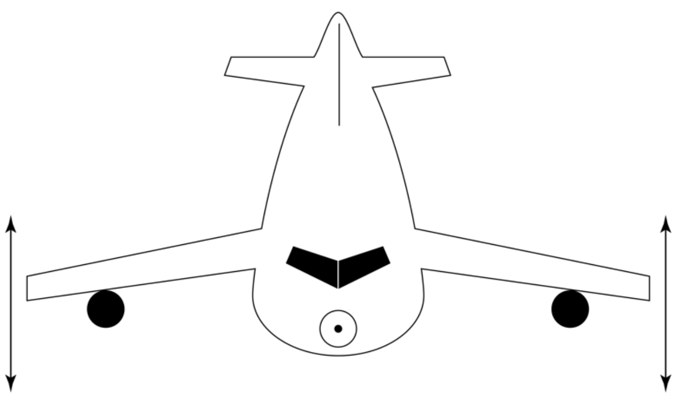
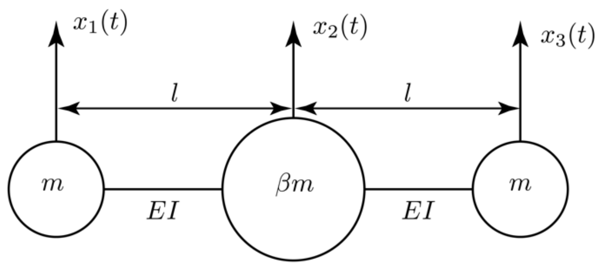
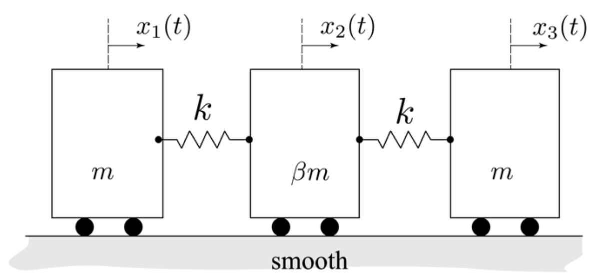
Homework Problem 3.1

The vibration in the vertical direction of an airplane and its wings can modeled as a three-degree-of-freedom system with one mass corresponding to the right wing (m), one mass for the left wing (m), an one mass for the fuselage (βm), where β is a constant greater than one. The stiffness connecting the three masses correspond to that of the wing and is a function of the modulus E of the wing. The generalized coordinates are the absolute positions $x_1(t)$, $x_2(t)$, and $x_3(t)$. The stiffness is $k = 3EI/l^3$ where E is the modulus of the wing and l is the length from the main body to fuselage. The figures show the wing vibration, the lumped mass/beam deflection model and the final spring mass model approximation of the beam model.







a) Use *Lagrange's Equations* to find the equations of motion of the spring mass model.
b) Calculate the natural frequencies an mode shapes of the system. One natural frequency is zero discuss the significance of this result in relation to the motion of the plane.
c) Calculate the total time response of the system and find the initial conditions on displacement such that 1) the system only responds as a rigid body) the system only respond at the 1st mode of vibration that is not a rigid body of vibration.

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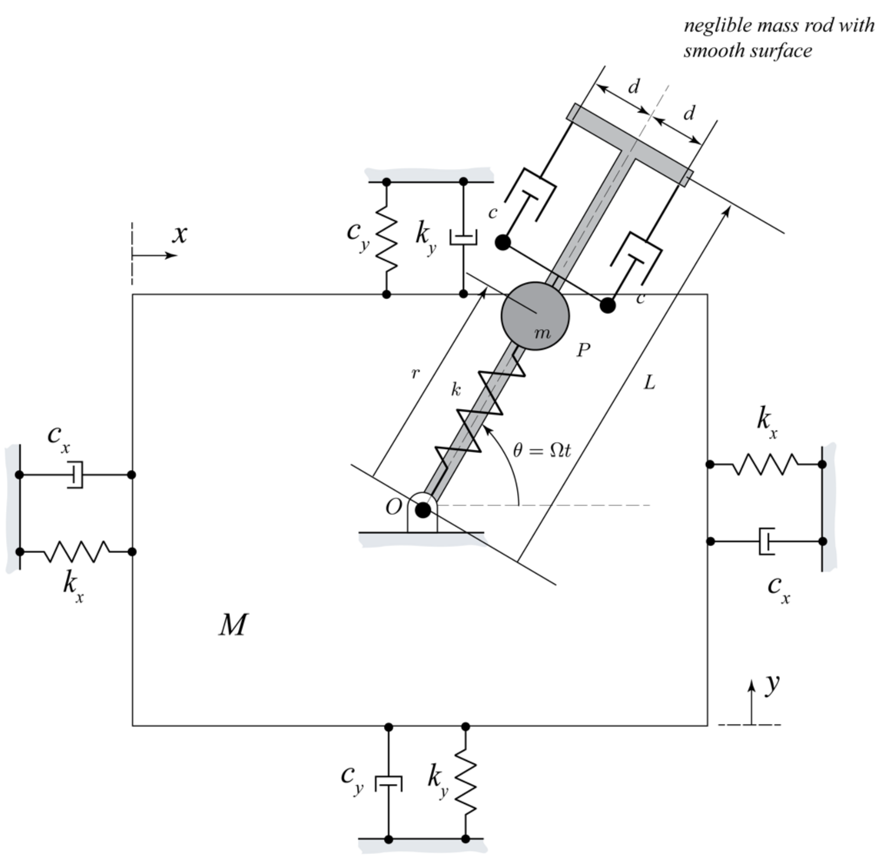
Homework Problem 3.2

Ground resonance is a potentially dangerous instability associated with rotary wing aircraft on flexible supports (landing gear). When the aircraft's rotor blades are disturbed to the point that the mass center of the rotor blade system is no longer in the center of the rotor, an inertial force is generated and passed via the rotor to the fuselage. The fuselage will begin to oscillate because of its ability to displace relative to the ground. One of two outcomes is possible depending on the system features. The vibration will be damped out if ground resonance does not exist. However, if the aircraft's stiffness and damping properties are such that ground resonance can occur, and if the rotor speed is within a critical band of frequencies, the displacement of the fuselage can cause the mass center of the rotor blades to move away from the rotor center. The oscillation of the fuselage increases as the inertial force increases. As a result, an unstable scenario exists in which the fuselage and blades interact, increasing the total amount of aircraft vibration.

The follow simplified system is used to model the ground resonance of a helicopter. It consists of a large mass *M*, flexibly connected to the ground, with a massless rod of length *L* rotating at a constant angular frequency Ω. A small mass *m* at *P* slides without friction on the rod where the motion of the small mass is restricted by a spring and damper. The small mass represents the aggregate effects of the blade.



Horizontal Plane

1) Use Lagrange's equation to develop the EOM's for this three-DOF system using generalized coordinates of *x*, *y*, and *r*. Recall that the potential energy stored in a spring is related to the square of the stretch in the spring, where the stretch is equal to the difference between its actual length, and the unstretched length. Also, in writing down the velocity of vector of *P*, you may want to review the polar kinematic expressions for velocity. Hint the velocity of P must be written in terms of two coordinate system a fixe XY coordinate system and the polar coordinate system. However, before you can apply Lagrange's equations the components of velocity must be transformed to a common coordinate system. Linearize the system note the linear system will have time dependent mass, damping, and stiffeness terms.

2) Show that there is an upper limit on the speed of rotation of the blades Ω, that once this frequency is passed the system will become unstable regardless of the other parameters in the model.

3) Partion the mass, stiffnes and damping matrces into constant and time dependent terms. Ignore the time dependent partions of the matrices and determine the natural frequencies and modeshapes of the system. Ignore damping.

4) Partion the mass, stiffnes and damping matrces into constant and time dependent terms. Determine the maximum values of the time dependent partions of the matrices and determine the natural frequencies and modeshapes of the system of unpartioned matrices using the maximum values. All the entries of the matices may not reach their maximum values at the same point. In this case you may have to look at different cases. In all cases ignore damping.

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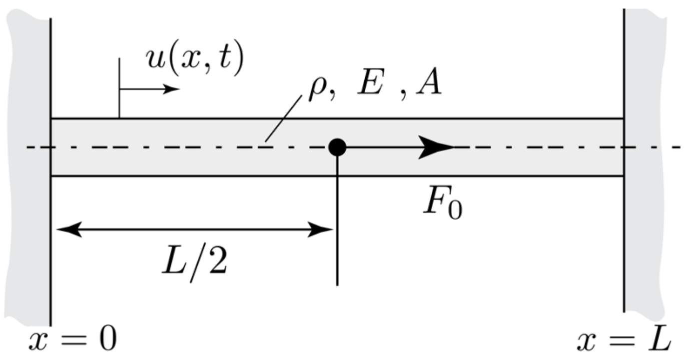
Homework Problem 3.3

A bar of uniform cross-sectional area A, length L, modulus of elasticity E, and density ρ is fixed at both ends. It is subjected to an axial force F_0 at the middle and is suddenly removed at $t = 0$. The force results in the following initial displacement field

$$u_o(x) = \begin{cases} \epsilon x, & for \quad 0 \le x \le L/2 \\ \epsilon(L - x), & for \quad L/2 \le x \le L' \end{cases}$$

where $\epsilon = F_0/(2AE)$.



Find the resulting vibration of the bar.

a) Write the partial differential governing equation of rod once the force is removed.

b) Derive the governing temporal and spatial ordinary differential equations that originate from assuming the solution to the equation of motion is separable in time and space.

c) Determine and solve the characteristic equation of the system.

d) Determine the modal functions of the system.

e) Use the initial condition to express the solution to the free vibration problem as a superposition of modes.