ME 563-Fall 2024

Homework No. 4

Due: November 13, 2024 11:59 pm on Gradescope

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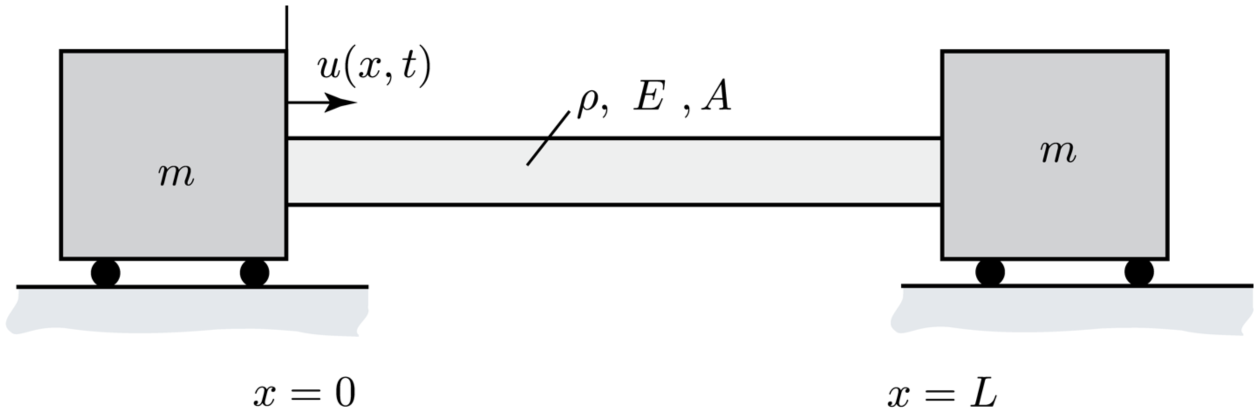
Homework Problem 4.1

The longitudinal motion of the rod shown above is to be described by $u(x,t)$.



a) Write down/derive the set of boundary conditions for this problem.

b) Derive the characteristic equation for this problem.

c) Make a hand sketch of the characteristic function in the equation in b) above. Indicate the first four roots of the characteristic equation in your sketch.

d) Solve the above characteristic equation for the first four natural frequencies. Use $m = \rho AL$.

e) Determine the modal functions (mode shapes) corresponding to four natural frequencies found above in d). Can the system admit rigid body modes?

f) Make hand sketches of the four mode shapes found in e). In these sketches, indicate the slopes of the mode shapes at $x = L$.

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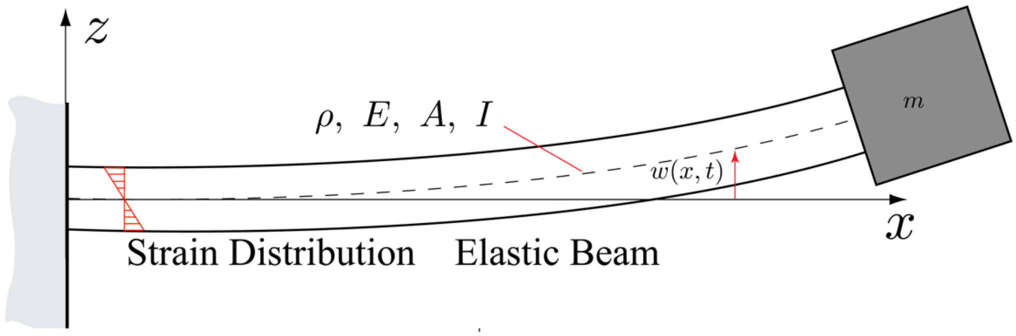
Homework Problem 4.2

The transverse motion of the beam shown above is to be described by $w(x,t)$.



a) Write down/derive the set of boundary conditions for this problem.

b) Derive the characteristic equation for this problem, use $m = \rho AL$.

c) Write a program to determine the first four natural frequencies of the system.

d) Plot the first four, mode shapes of the beam

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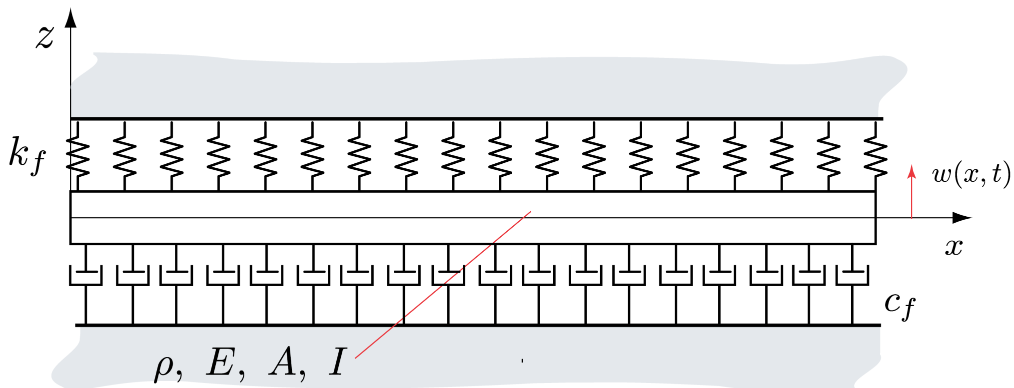
Homework Problem 4.3

A **free-free beam** rest on a foundation that is viscously damped. The foundation stiffness and damping have negligible effect on boundary conditions of the beam. The foundation has a stiffness per unit length of k_f and a damping coefficient per unit length of c_f. The beam is initially at rest.



a) Derive the differential equation of motion (EOM) for the system. You may start the derivation from the standard form of the fourth order partial differential equation that describes the bending of a beam and treat the foundation as a distributed force.

b) Assuming a spatiotemporal solution for $w(x, t)= W(x)T(t)$, derive the governing temporal and spatial ordinary differential equations that originate from assuming the solution to the equation of motion is separable in time and space. State boundary conditions for the boundary value problem. DO NOT PROCEED FURTHER

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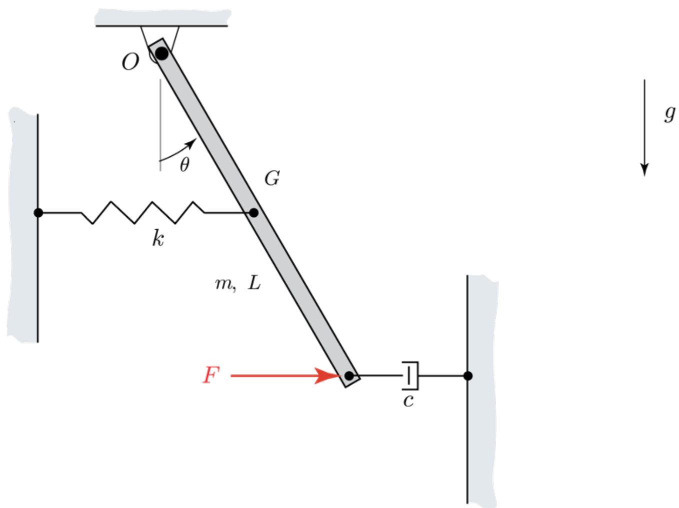
Homework Problem 4.4

Consider the pendulum mechanism with a pinned support at point O. The bars has length L and m. A damper is attached at point C and the spring is attached at G. G is the center of mass of the beam. A force $F(t) = F_o \sin \Omega t$ is applied at point C.



a) Derive the differential equation of motion (EOM) for the system.
b) What is the damped and undamped natural frequency of the system?
c) Derive an expression for the steady state amplitude of the system.