ME 563-Fall 2024

Homework No. 5

Due: November 26, 2024 11:59 pm on Gradescope

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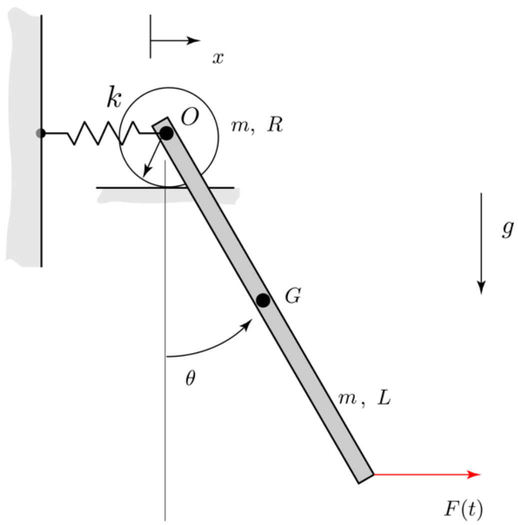
Homework Problem 5.1

A forcing $F(t) = f_0 \sin\Omega t$ acts at the end of the thin, homogeneous bar of the two-DOF system shown below. The wheel can be modeled as a cylinder with mass m, radius R and rolls without slipping. The response of the system is to be described by the coordinates $x(t)$ and $\theta(t)$. Let $g/L = 2k/m$.



a) Derive the equations of motion for the system

b) Derive the particular solutions $x_p(t)$ and $\theta_p(t)$ for the system.

c) At what values of the temporal frequency Ω does resonance occur in the system?

d) Show that the "shape" (ratio of amplitudes) of the response is that of the first mode when excited at the first natural frequency, and that the shape is that of the second mode when excited at the second natural frequency.-

e) At what values (if any) of the temporal frequency Ω do anti-resonances occur for $x_p(t)$? For $\theta_p(t)$?

f) Make plots for the amplitudes of $x_p(t)$ and $\theta_p(t)$ vs. the temporal frequency Ω.

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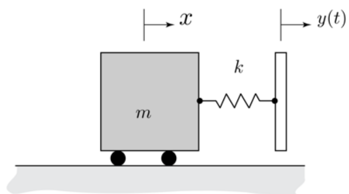
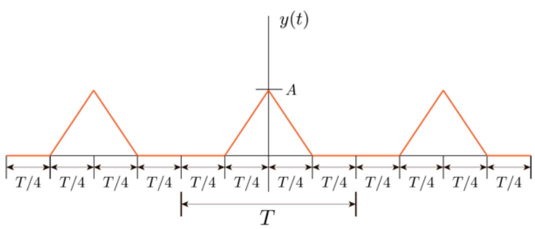
Homework Problem 5.2





The undamped, single-DOF system above is given a base excitation of $y(t)$. The base motion $y(t)$ is T-periodic in time, as shown above.

a) Determine the Fourier series of $y(t)$.

b) Use Matlab to make a plot of $y(t)$ vs. t/T for your Fourier series. Use a sufficient number of terms in your Fourier series when making the plot to insure that the series has converged.

c) Derive the equation of motion for the system.

d) Determine the particular solution for the response, $x_P(t)$.

e) Use Matlab to make a plot of $x_P(t)$ vs. t/T for your Fourier series corresponding to $T = 0.87\ T_n$, where $T_n = 2\pi\ (m\ /\ k)^{1/2}$ is the natural period of oscillation for the system. Use the same number of terms in your response $x_P(t)$ as you did in your Fourier series for $y(t)$ above.

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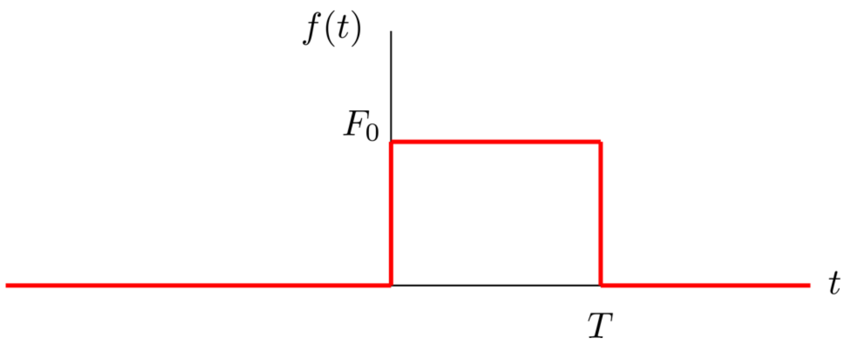
Homework Problem 5.3



Consider a damped, single-DOF system with an equation of motion of:
$m\ddot{x} + c\dot{x} + kx = F(t)$
It was shown in lecture that the convolution integral for the undamped case ($\zeta = 0$) with zero initial conditions
$x(0) = \dot{x}(0) = 0$
that the convolution integral could be written as:
$$x(t) = \int_0^t h(t-\tau)f(\tau)d\tau$$
where the impulse response function was given by:
$$h(t-\tau) = \frac{1}{m\omega_n}\sin\omega_n(t-\tau).$$
a) Show that the convolution integral solution for zero initial conditions for the critically-damped case ($\zeta = 1$) can be written in the same general form:
$$x(t) = \int_0^t h(t-\tau)f(\tau)d\tau$$
What is the impulse response function $h(t-\tau)$ in this case? Feel free to start your derivation using the general form of the convolution integral found in the lecturebook in terms of the fundamental solutions $u(t)$ and $v(t)$.

b) Use the convolution integral derived above in a) to determine the *response of the system* to the forcing shown above, where $T = 0.50\ T_n$, where $T_n = 2\pi\ (m\ /\ k)^{1/2}$ is the natural period of oscillation for the system.