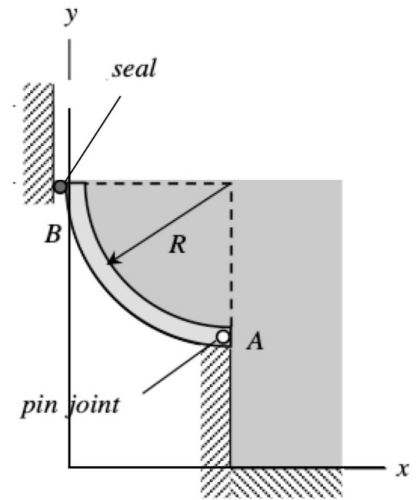**Given**: A water gate, shaped as a quarter-circle arc, has a width of b (out of the paper). The gate is pinned to a fixed support at A and with end B pressed against a smooth seal. The density of the water is ρ. Consider the weight of the gate to be negligible compared to the weight of the water above the gate.

Find: Following the four steps below, determine the reactions on the gate at pin joint A and at seal B.

seal

B   *R*   *A*

pin joint

Step 1 – FBD: Draw the FBD of the gate with the water above the gate in the figure provided to the right.

Step 2 – Equilibrium: Write down the equilibrium equations for your FBD.
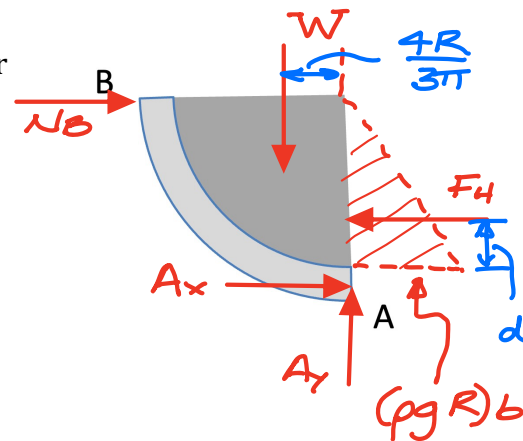

(1) $\sum F_x = N_B + A_x - F_H = 0$

(2) $\sum F_y = -W + A_y = 0$

(3) $\sum M_A = W\left(\frac{4R}{3\pi}\right) - N_B R + F_H d = 0$

w/ $W = \left(\frac{1}{4}\pi R^2\right)(b)\rho g$

$F_H = \frac{1}{2}(\rho g R b)R$

$d = \frac{1}{3}R$

Step 3 – Solvability: Count the number equations and the number of unknowns.

3 eqns / 3 unknowns (A_x, A_y, N_B)

Step 4 – Solve: Determine the reactions on the gate at A and B. Leave in terms of, at most, R, ρ, b and g.

(2) $\Rightarrow$ $A_y = W = \frac{\pi}{4}R^2 b \rho g$

(3) $\Rightarrow$ $N_B = \frac{1}{R}\left[\frac{4R}{3\pi}\left(\frac{\pi}{4}R^2 b \rho g\right) + \frac{R}{3}\left(\frac{1}{2}R^2 b \rho g\right)\right]$

$\qquad = \frac{1}{2}R^2 b \rho g$

(1) $\Rightarrow$ $A_x = F_H - N_B = \frac{1}{2}R^2 b \rho g - \frac{1}{2}R^2 b \rho g = 0$