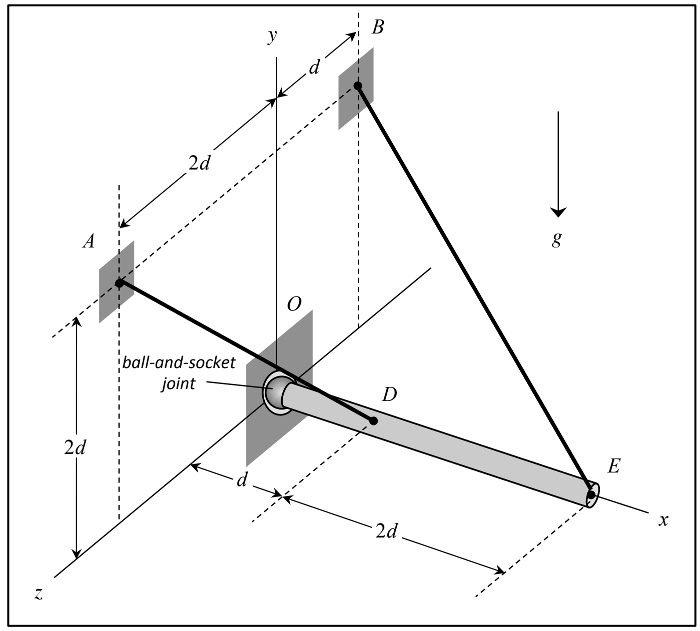
Final Examination **SOLUTION**

ME 270 – Summer 2024 - Prague

PROBLEM NO. 1 (20 points)



Given: A homogeneous boom pole, having a weight of W, is supported by a ball-and-socket joint at end O, and by two cables, AD and BE.

Find: Using the four steps outlined below, determine the tension force acting on the pole by each cable.

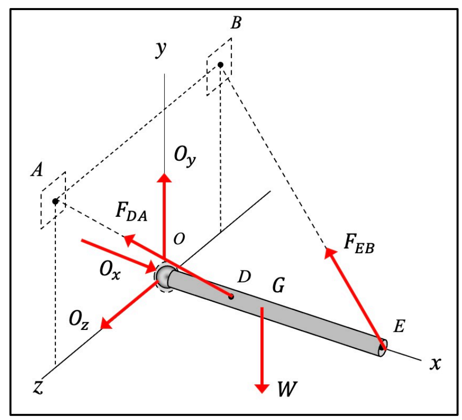
Solution:

Step 1: FBD

Draw an FBD of bar OE in the diagram provided below right.

Step 2: Equilibrium

Write down the force vectors on OE due to cables AD and BE in terms of their *xyz*-components and in terms of the unknown tensions F_{AD} and F_{BE}. Write down the equilibrium equations that are necessary for determining the two tension forces.



Tension force vectors:

$$\vec{F}_{EB} = F_{EB}\hat{u}_{EB}$$
$$= F_{EB}\left(\frac{-3d\hat{\imath} + 2d\hat{\jmath} - d\hat{k}}{\sqrt{(-3d)^2 + (2d)^2 + (-d)^2}}\right)$$
$$= F_{EB}\left(-\frac{3}{\sqrt{14}}\hat{\imath} + \frac{2}{\sqrt{14}}\hat{\jmath} - \frac{1}{\sqrt{14}}\hat{k}\right)$$

$$\vec{F}_{DA} = F_{DA}\hat{u}_{DA}$$
$$= F_{DA}\left(\frac{-d\hat{\imath} + 2d\hat{\jmath} + 2d\hat{k}}{\sqrt{(-d)^2 + (2d)^2 + (2d)^2}}\right) = F_{DA}\left(-\frac{1}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}\right)$$

Equilibrium:

$$\sum\vec{M}_O = \vec{r}_{OE} \times \vec{F}_{EB} + \vec{r}_{OG} \times \vec{W} + \vec{r}_{OD} \times \vec{F}_{DA}$$
$$\vec{0} = (3d\hat{\imath}) \times F_{EB}\left(-\frac{3}{\sqrt{14}}\hat{\imath} + \frac{2}{\sqrt{14}}\hat{\jmath} - \frac{1}{\sqrt{14}}\hat{k}\right) + \left(\frac{3d}{2}\hat{\imath}\right) \times (-W\hat{\jmath}) + (d\hat{\imath}) \times F_{DA}\left(-\frac{1}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}\right)$$
$$= d\left(\frac{3F_{EB}}{\sqrt{14}} - \frac{2F_{DA}}{3}\right)\hat{\jmath} + d\left(\frac{6F_{EB}}{\sqrt{14}} - \frac{3}{2}W + \frac{2F_{DA}}{3}\right)\hat{k}$$

$$(1)\quad \hat{\jmath}:\quad \frac{3F_{EB}}{\sqrt{14}} - \frac{2F_{DA}}{3} = 0$$

$$(2)\quad \hat{k}:\quad \frac{6F_{EB}}{\sqrt{14}} - \frac{3}{2}W + \frac{2F_{DA}}{3} = 0$$

Step 3: Solvability

Are your equilibrium equations solvable? Can you solve these equations? Explain.
2 equations and 2 unknowns (F_{EB} and F_{DA}). Can solve.

Step 4: Solve

Determine the cable tension forces acting on the boom pole. Write your answers in terms of W and as vectors. Please use *unit vector notation*.

$$(1) \quad \Rightarrow \quad \frac{3F_{EB}}{\sqrt{14}} - \frac{2F_{DA}}{3} \quad \Rightarrow \quad F_{DA} = \frac{9F_{EB}}{2\sqrt{14}}$$

$$(2) \quad \Rightarrow \quad \frac{6F_{EB}}{\sqrt{14}} - \frac{3}{2}W + \frac{3F_{EB}}{\sqrt{14}} = 0 \quad \Rightarrow \quad \boxed{F_{EB} = \frac{\sqrt{14}}{6}\,W} \Rightarrow \boxed{F_{DA} = \frac{3}{4}W}$$

Therefore:

$$\boxed{\begin{aligned} \vec{F}_{EB} &= W\left(-\frac{1}{2}\hat{\imath} + \frac{1}{3}\hat{\jmath} - \frac{1}{6}\hat{k}\right) \\ \vec{F}_{DA} &= W\left(-\frac{1}{4}\hat{\imath} + \frac{1}{2}\hat{\jmath} + \frac{1}{2}\hat{k}\right) \end{aligned}}$$

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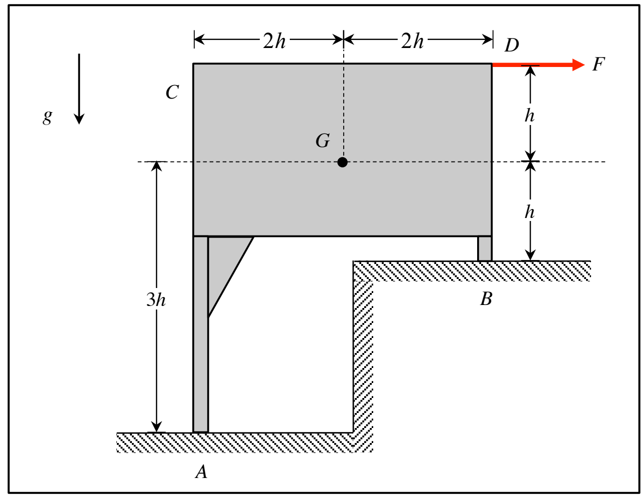
PROBLEM NO. 2 (20 points)



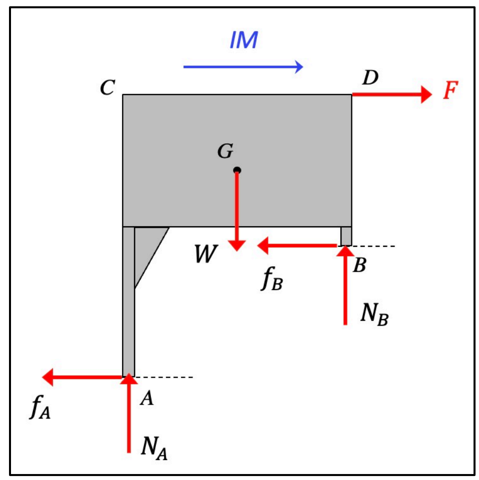
Given: A block, having a weight of W and with its center of mass at G, is supported by light-weight legs at A and B. The coefficient of static friction of $\mu_S = 0.6$ exists between the legs and ground. A horizontal force F acts on the block at D.

Find: Following the four steps below, determine the largest value of F that can be applied and not have the block move. State whether this load corresponds to impending tipping or impending slipping

Solution

Step 1: FBD

Draw an FBD of the block that is valid for a general state of equilibrium, and not specific to either tipping or slipping. Use the figure provided to the right.



Step 2: Equilibrium

Based on your FBD above, write down the equilibrium equations that describe the general equilibrium of the block.

(1) $\sum M_B = -F(2h) + W(2h) - N_A(4h) - f_A(2h) = 0$

(2) $\sum F_x = F - f_A - f_B = 0$

(3) $\sum F_y = -W + N_A + N_B = 0$

Steps 3a and 4a: Solvability and solve for SLIPPING
Write down the additional equation(s) that describe the conditions for impending slipping of the block. Cite the solvability conditions, and solve for the corresponding loading F for slipping. Leave your answer in terms of W.
For slipping:

$(4a)\ \ f_A = \mu_S N_A$

$(5a)\ \ f_B = \mu_S N_B$

This gives 5 equations and 5 unknowns. Can solve.

$(3)\ \Rightarrow\quad N_A + N_B = W$

$(2), (4a)$ and $(5a)\ \ \Rightarrow\quad F_{slip} = \mu_S N_A + \mu_S N_B = \mu_S W$

Steps 3b and 4b: Solvability and solve for TIPPING
Write down the additional equation(s) that describe the conditions for impending tipping of the block. Cite the solvability conditions, and solve for the corresponding loading F for tipping. Leave your answer in terms of W.
For tipping:

$(4b)\ \ f_A = 0$

$(5b)\ \ N_A = 0$

This gives 5 equations and 5 unknowns. Can solve.

(1) and $(4b)\ \ \Rightarrow\quad -F(2h) + W(2h) = 0\quad \Rightarrow\quad F_{tip} = W$

Final result:
Based on your answers above, determine the maximum load F allowed for the block to not move. Express your answer in terms of W. Does this motion correspond to slipping or tipping? Also, provide values for the friction and normal contact forces at A and B at this loading, with these forces also being written in terms of W.

We need to choose the smaller of the two forces. Therefore, the maximum load F for which the block does not move is: $F_{max} = F_{slip} = 0.6W$.

Therefore:

$$(1)\ \Rightarrow\ -2F_{slip} + 2W - 2N_A(2+\mu_S) = 0\ \Rightarrow\ \ N_A = W\left(\frac{1-\mu_S}{2+\mu_S}\right) = W\left(\frac{1-{}^3/_5}{2+{}^3/_5}\right) = \frac{2}{13}\,W$$

$$(3)\ \Rightarrow\quad N_B = W - N_A = \frac{11}{13}W$$

$$(4a)\ \Rightarrow\ \ f_A = \mu_S N_A = \frac{6}{65}W$$

$$(5a)\ \Rightarrow\ \ f_B = \mu_S N_B = \frac{33}{65}W$$

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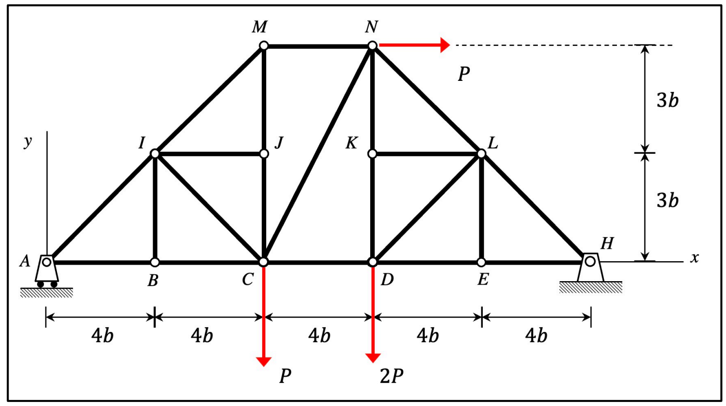
PROBLEM NO. 3 (20 points)

Given: Consider the truss shown below with the loading on joints C, D and N. Each member has a cross-sectional area of A.

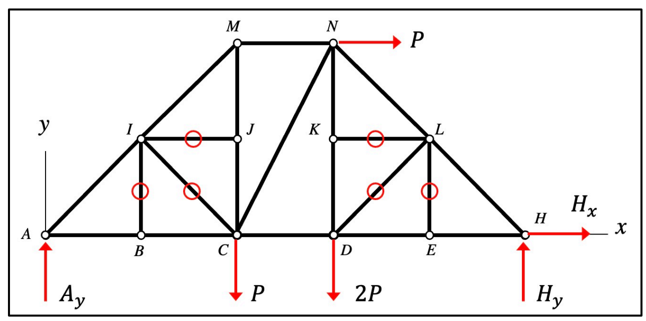
Find: For this problem:



a) Determine the external reactions acting on the truss at supports A and H.
b) Identify all zero-force members in the truss.
c) Using the method of sections, determine the load carried by members BC, CJ and MN. Identify each member as either being in tension, in compression or carrying zero load.
d) Determine the stress in member MN. Is this stress tensile or compressive?

Express your answers in c) and d) in terms of, at most: P and A.

a) External reactions



$$\sum M_A = H_y(20b) - P(8b) - 2P(12b) - P(6b) = 0$$

$$\Rightarrow \boxed{H_y = \frac{19}{10}P}$$

$$\sum F_x = P + H_x = 0 \quad \Rightarrow \quad \boxed{H_x = -P}$$

$$\sum F_y = A_y - P - 2P + H_y$$

$$\Rightarrow \quad \boxed{A_y = 3P - H_y = 3P - \frac{19}{10}P = \frac{11}{10}P}$$

b) Zero-force members

Joint B: $F_{BI} = 0$

Joint J: $F_{IJ} = 0$

Joint I: $F_{CI} = 0$

Joint E: $F_{EL} = 0$

Joint K: $F_{KL} = 0$

Joint L: $F_{DL} = 0$

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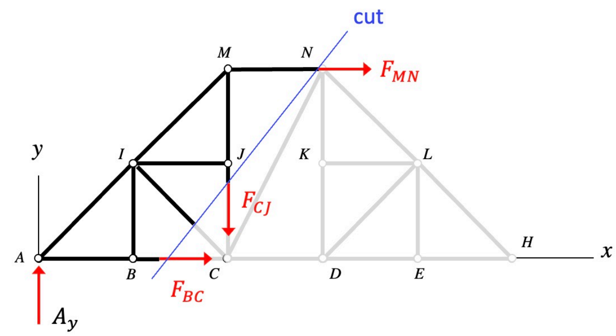
PROBLEM NO. 3 (continued)



c) Internal loads

Make a cut through members BC, CI, CJ and MN, and recall that CI is a zero-force member. All member forces drawn as if the member were in tension.

(1) $\sum M_C = -A_y(8b) - F_{MN}(6b) = 0$

(2) $\sum F_x = F_{MN} + F_{BC} = 0$

(3) $\sum F_y = A_y - F_{CJ} = 0$

Three equations and three unknowns. Solve:

$$(1) \;\Rightarrow\; F_{MN} = -\frac{4}{3}A_y = -\frac{22}{15}P \quad (compression)$$

$$(2) \;\Rightarrow\; F_{BC} = -F_{MN} = \frac{22}{15}P \quad (tension)$$

$$(3) \;\Rightarrow\; F_{CJ} = A_y = \frac{11}{10}P \quad (tension)$$

d) Stress in member MN

$$\sigma_{MN} = \frac{F_{MN}}{A} = -\frac{22}{15}\frac{P}{A} \quad (compressive)$$

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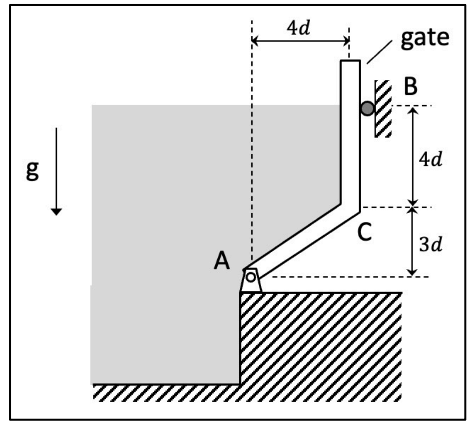
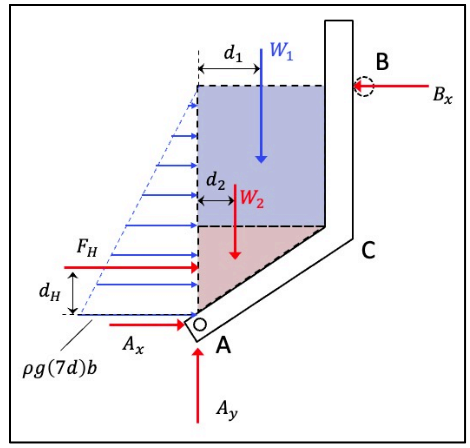
PROBLEM NO. 4 (20 points)



Given: Gate ACB holds back a pool of static water, with the gate pinned to ground at end A and supported by a roller support at B. The gate has a constant depth dimension of b into the page, and the water has a mass density of ρ. The weight of the gate can be considered to be negligible compared to the loads of the gate by the water, and consider the thickness of the gate to be small compared to the other dimensions.

Find: Using the four steps below, you are asked here to determine the load exerted on the gate by the roller support B.



Solution

Step 1: FBD

Draw an FBD of the gate along with the water that exists above the gate. Clearly identify the magnitude and location of the water-related forces in the FBD in terms of ρ, g, d and b.

$W_1 = \rho g(4d)^2 b = 16\rho g d^2 b$
$W_2 = \rho g\,(1/2)(4d)(3d)b = 6\rho g d^2 b$
$F_H = (1/2)(7\rho g d b)(7d) = (49/2)\rho g d^2 b$
$d_1 = 2d$
$d_2 = 4d\,/\,3$
$d_H = 7d\,/\,3$

Step 2: Equilibrium

Write down the equilibrium equation(s) needed to determine the reaction on the gate at B.

$$\sum M_A = -F_H d_H - W_1 d_1 - W_2 d_2 + B_x(7d) = 0$$

Step 3: Solvability

Count the number of available equations and the number of unknowns. Can you solve these equations? Explain.

1 equation and 1 unknown – can solve

Step 4: Solve

Determine the reaction force on the gate at support B. Write your answer in terms of, at most: ρ, g, d and b.

$$\begin{aligned} B_x &= (1/7d)(F_H d_H + W_1 d_1 + W_2 d_2) \\ &= (\rho g d^2 b/7)[(49/2)(7/3) + 16(2) + (6)(4/3)] \\ &= (583/42)\rho g d^2 b \end{aligned}$$

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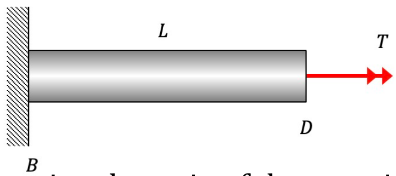
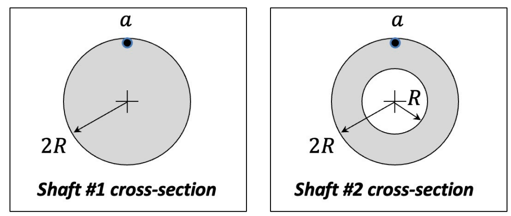
PROBLEM NO. 5

PART A (4 points)

Given: Consider two shafts having the same generic description shown below of having a length L, being made of a material having a shear modulus G, and with an axial torque of T applied. Shaft #1 has a solid cross-section of radius $2R$, whereas Shaft #2 is tubular with inner and outer radii of R and $2R$, respectively. Let $|\tau_a|_1$ and $|\tau_a|_2$ represent the magnitudes of the shear stress at points "a" on cross-sections 1 and 2, respectively.





Find: Determine the ratio of the magnitudes of the shear stresses at "a" for cross-sections #1 and #2: $|\tau_a|_2 \,/\, |\tau_a|_1$. Express your answer in terms of, at most: T, R, G and L.

Shaft #1: $|\tau_a|_1 = \frac{T(2R)}{J} = \frac{2TR}{\pi(2R)^4/2} = \frac{TR}{4\pi R^4}$

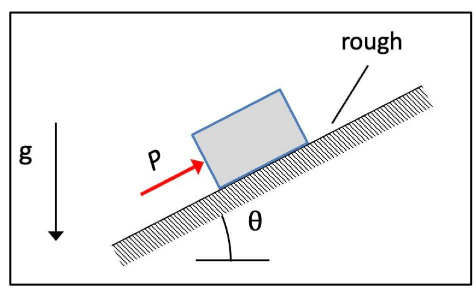
Shaft #2: $|\tau_a|_2 = \frac{T(2R)}{J} = \frac{2TR}{\pi(2R)^4/2 - \pi R^4/2} = \frac{4TR}{15\pi R^4}$

Therefore:

$$\frac{|\tau_a|_2}{|\tau_a|_1} = \frac{16}{15}$$

PART B (4 points)

Given: A block rests on a rough incline with a force P acting on the block in the direction of up the incline. A force of friction f acts on the block through its contact with the incline.



Find: It is desired to determine the minimum P (P_{min}) that can act on the block without allowing the block to move. Circle the correct response below that most accurately describes the *direction* of the friction force f on the block for this applied load. Provide a short justification for your response.

a) f on the block points UP the incline.

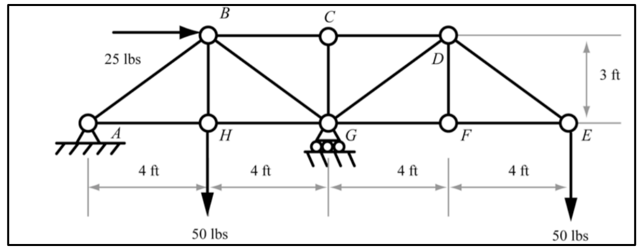
b) f on the block points DOWN the incline.

c) f = 0

d) More information is needed to answer this question.

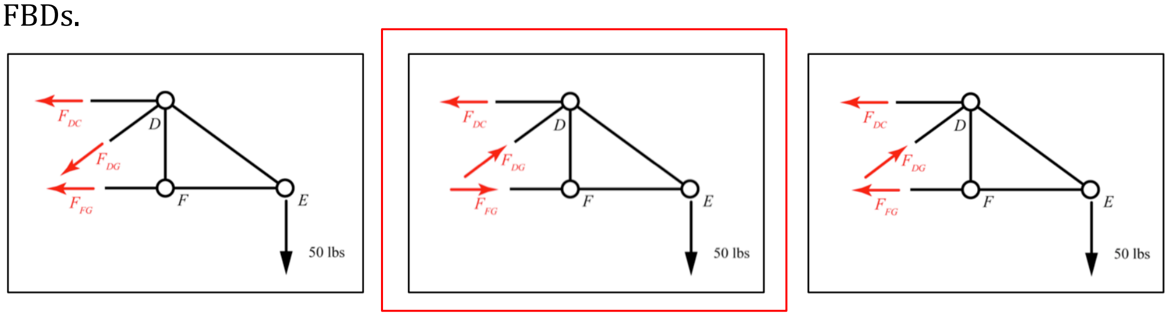
Since the IM of the block is DOWN the incline, f opposes this and acts UP the incline.

PART C (4 points)



Given: Consider the loading on the truss shown below. The method of sections has been used to determine the loads carried by members DG, FG and DC.

Find: Circle the FBD below that correctly represents the loading state (tension or compression) of members DG, DF and DC. Provide a brief explanation for your response. *HINT*: Consider the moment about joint D due to the forces acting on the FBDs.



Using the summation of moments about joint D, we see that F_{FG} must act to the RIGHT to maintain equilibrium. (Also, using the summation of forces in the y-direction, we see that F_{DG} must point in the UPWARD direction.)

PART D (4 points)

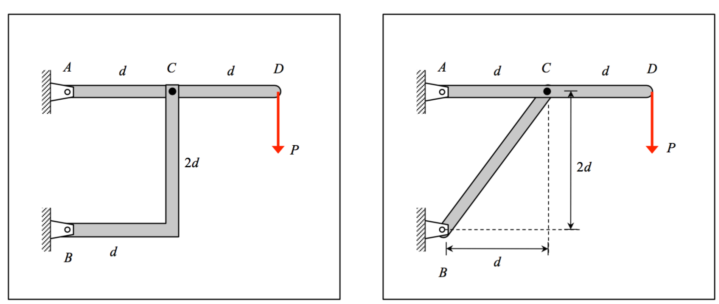
Given: Consider Frames 1 and 2 below. Let $\left|(F_C)_1\right|$ and $\left|(F_C)_2\right|$ represent the magnitudes of the forces of pin C on member AD in Frames 1 and 2, respectively.

Find: Circle the correct response below regarding the relative sizes of $\left|(F_C)_1\right|$ and $\left|(F_C)_2\right|$ Provide a brief explanation for your response.

a) $\left|(F_C)_1\right| > \left|(F_C)_2\right|$

b) $\left|(F_C)_1\right| = \left|(F_C)_2\right|$

c) $\left|(F_C)_1\right| < \left|(F_C)_2\right|$

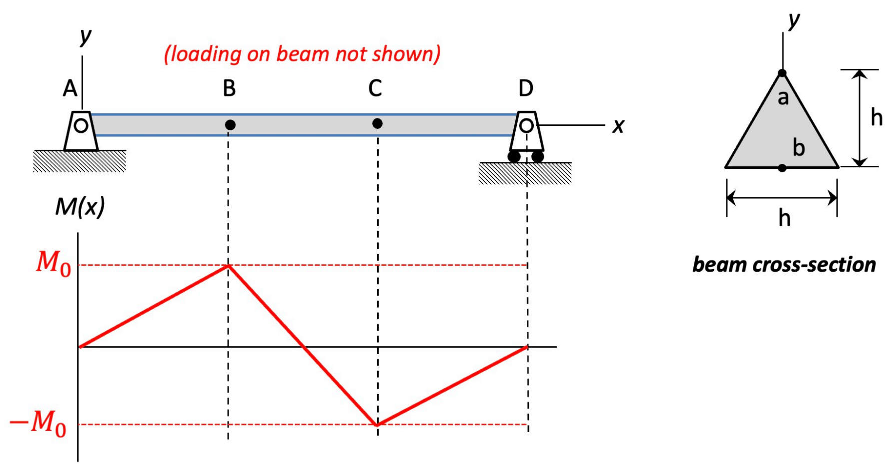


Frame 1 **Frame 2**

Since BC is a two-force member for both frames, the FBDs for the two frames will be the same. Therefore, the force on pin C will be the same for each frame.

PART E (4 points)

Given: Consider the simply-supported beam AD, having the triangular cross-section shown. The loading on the beam (not shown) produces the bending moment diagram provided below.



Find: For this loading, determine:

a) the location(s) (along the beam and on the cross-section) of maximum *compressive* normal stress in the beam.

b) the location (s) (along the beam and on the cross-section) of maximum *tensile* normal stress in the beam.

<u>*Location B*</u>

$$\sigma_a = -\frac{M_0(2h/3)}{I} \quad ; \quad COMPRESSION \leftarrow \quad largest\ compressive\ stress$$

$$\sigma_b = \frac{M_0(h/3)}{I} \quad ; \quad TENSION$$

<u>*Location C*</u>

$$\sigma_a = \frac{M_0(2h/3)}{I} \quad ; \quad TENSION \leftarrow \quad largest\ tensile\ stress$$

$$\sigma_b = -\frac{M_0(h/3)}{I} \quad ; \quad COMPRESSION$$