ME 270 – Fall 2024 Exam 1 NAME (Last, First): SOLUTION (Please Print)

PUID #: _____(Please Print)

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: _____

Instructor's Name and Section: (Circle Your Section)

Sections:

J. Jones, Section 001, MWF 9:30AM-10:20AM
S. Dyke, Section 003, MWF 10:30AM-11:20AM
J. Jones, Section 002, MWF 11:30AM-12:20PM
F. Semperlotti, Section 005, MWF 12:30PM-1:20PM
F. Zhao, Section 008, MWF 1:30PM-2:20PM
F. Semperlotti, Section 009, MWF 2:30PM-3:20PM
L. Krest, Section 010, MWF 3:30PM-4:20PM
M. Murphy, Section 007, TR 9:00AM-10:15AM
J. Jones, Section Y01, Distance Learning

Please review and sign the following statement:

Purdue Honor Pledge – "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

Signature: _____

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

- The allowable exam time for Exam 1 is 90 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the back side of your exam paper.

If the solution does not follow a logical thought process, it will be assumed in error.

When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of the cover page. Also, be sure to identify the page numbers for each problem before final submission on Gradescope. Do not include the cover page or the equation sheet with any of the problems.

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PROBLEM 1 (20 points)

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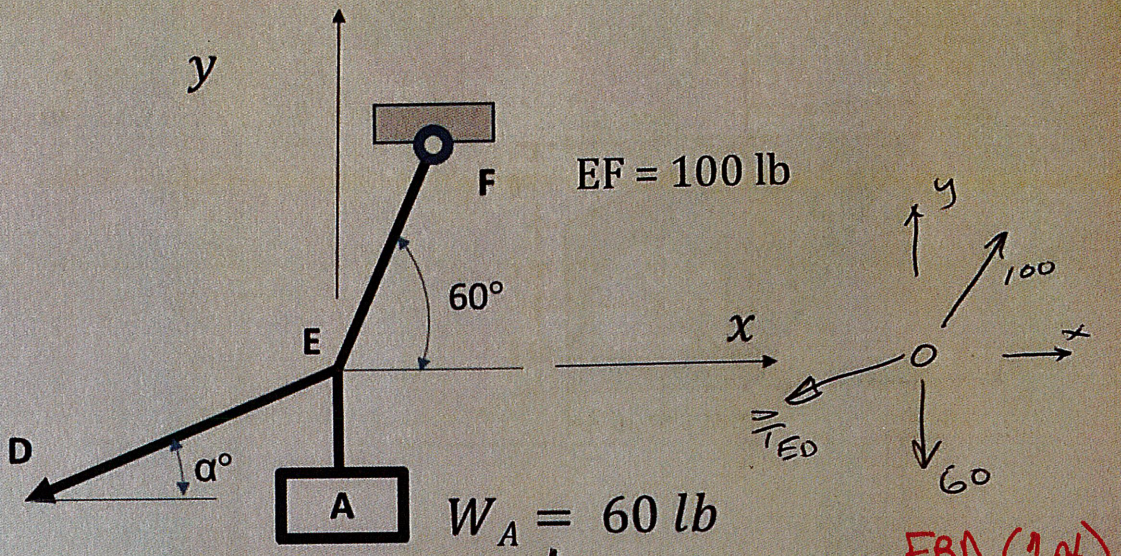
1A.

1B. Block A weighing 60 lbs is held in static equilibrium by two cables (EF and ED). The tension in cable EF is 100lbs. Using the figure below determine the angle, α and the tension in cable ED required to maintain static equilibrium. **(5 pts)**



FBD (1 pt.)

$\Sigma F_x = 0 = 50 - T_{DE_x}$

$T_{DE_x} = 50$

$\Sigma F_y = 0 = 86.6 - 60 - T_{ED_y}$

$T_{ED_y} = 26.6$

$|\vec{T}_{ED}| = \sqrt{50^2 + 26.6^2} = 56.63$

Alpha = $\tan^{-1}(26.6/50)$ = 28.0°

Or

Alpha = $\cos^{-1}(50/56.63)$ = 28.0°

(2pts)

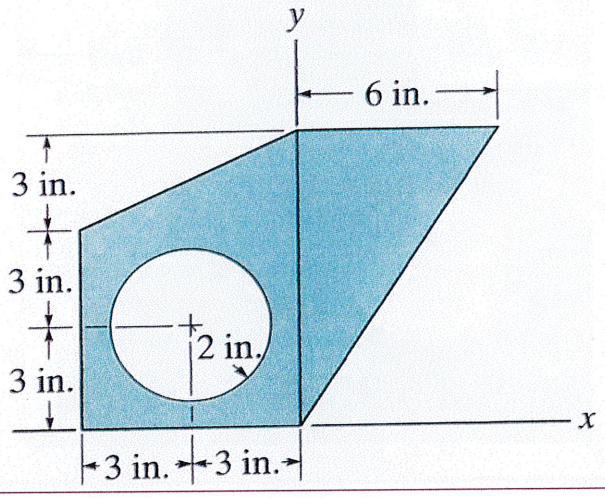
α =	28.0°	(3pts)
$\|\vec{T}_{ED}\|$ =	56.63 lbs	(2pts)

1B.

1C. Using the method of composite parts, **find** the area and the x-centroid (x_c) of the shaded area in the figure below with respect to the coordinate axes provided. Please show your work to receive credit. If the circular hole was filled in, qualitatively what impact would this have on the magnitude of x_c and y_c (i.e., ignore the signs). (No calculations are required). **(5 pts)**

Shape	Area (in^2)	x_c (in)
Square	$36\ in^2$	-3 in
Triangle 1	$9\ in^2$	-2 in
Circle	$-4\pi\ in^2$ ($-12.57 in^2$)	-3 in
Triangle 2	$27\ in^2$	$+2$ in

Area Total = $59.4 in^2$



$$(\text{Area Total})\ x_c = A_1 x_{c_1} + A_2 x_{c_2} + A_3 x_{c_3} + A_4 x_{c_4}$$

$$x_c = \frac{A_1 x_{c_1} + A_2 x_{c_2} + A_3 x_{c_3} + A_4 x_{c_4}}{\text{Area Total}}$$

$$x_c = \frac{(36 in^2)(-3 in) + (9 in^2)(-2 in) + (-12.57 in^2)(-3 in) + (27 in^2)(2 in)}{59.4 in^2}$$

$$\therefore x_c = -0.577 in$$

A = 59.4 in^2 (1 pt) x_c = −0.577 in (2 pts)

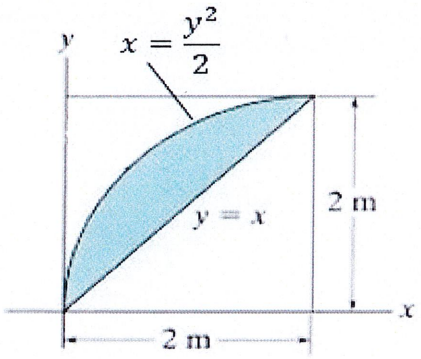
x_c = Increase Stay the Same **Decrease** (Circle One) (1 pt)

y_c = Increase Stay the Same **Decrease** (Circle One) (1 pt)

1C.

1D. Using the method of integration, determine the area and the x-centroid (x_c) of the shaded area with respect to the coordinate axes provided. Qualitatively, would you expect for y_c to be larger, smaller or equal to x_c? (No calculations are required). **(5 pts)**

$$A = \int_{y_1=0}^{y_2=2} \int_{x_1=\frac{y^2}{2}}^{x_2=y} dx\,dy = \int_0^2 [x]_{\frac{y^2}{2}}^{y} dy$$



$$A = \int_0^2 \left(y - \frac{y^2}{2}\right) dy = \left[\frac{y^2}{2} - \frac{y^3}{6}\right]_0^2$$

$$\therefore A = \frac{4}{2} - \frac{8}{6} = 2 - 1\tfrac{1}{3} = \boxed{\frac{2}{3}\,m^2 = 0.667\,m^2}$$

$$A(x_c) = \int_{y_1=0}^{y_2=2} \int_{x_1=\frac{y^2}{2}}^{x_2=y} x\,dx\,dy = \int_0^2 \left[\frac{x^2}{2}\right]_{\frac{y^2}{2}}^{y} dy = \int_0^2 \left(\frac{y^2}{2} - \frac{y^4}{8}\right) dy$$

$$(2/3\,m^2)\,x_c = \left[\frac{y^3}{6} - \frac{y^5}{40}\right]_0^2 = \frac{8}{6} - \frac{32}{40} = \frac{4}{3} - \frac{4}{5} = \frac{20-12}{15} = \frac{8}{15}$$

$$\therefore x_c = \left(\frac{8}{15}\right)\left(\frac{3}{2}\right) = \boxed{\frac{4}{5}\,m = 0.8\,m}$$

A =	$\frac{2}{3}\,m^2$ or 0.667 m^2	(1 pt)	x_c =	$\frac{4}{5}\,m$ or 0.8	m (3 pts)

y_c Trend: (**$y_c > x_c$**) $y_c < x_c$ $y_c = x_c$ (Circle One) (1 pt)

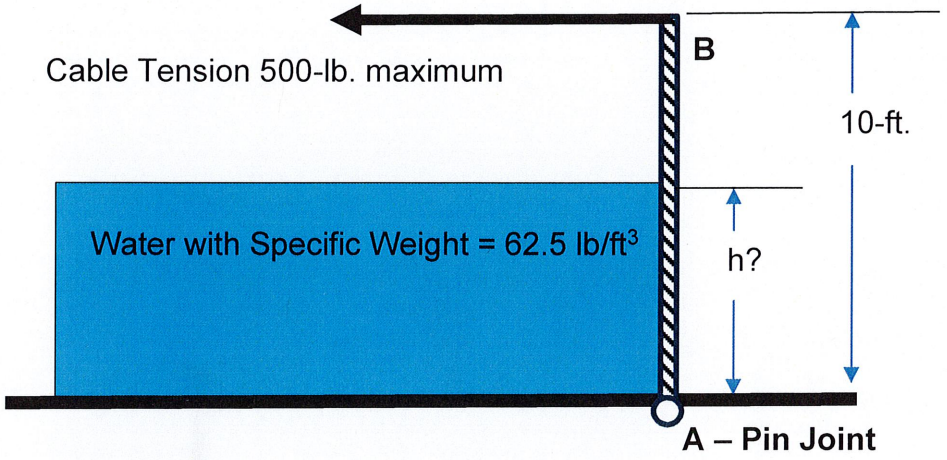
PROBLEM 1 (20 points)

1D. **1A.** A cable is attached at point **B** that is **10 feet** above a pin joint that holds a **1 foot wide gate** shut to prevent spillage of the water. The specific weight of the water is **62.5 lb/ft^3**. The cable can reliably apply 500-lb. load without breaking.

Please Determine:

i. The Equivalent hydrostatic force applied to the 1 foot wide gate in terms of h.

ii. The maximum height the water can attain before the force in the cable is 500-lb.



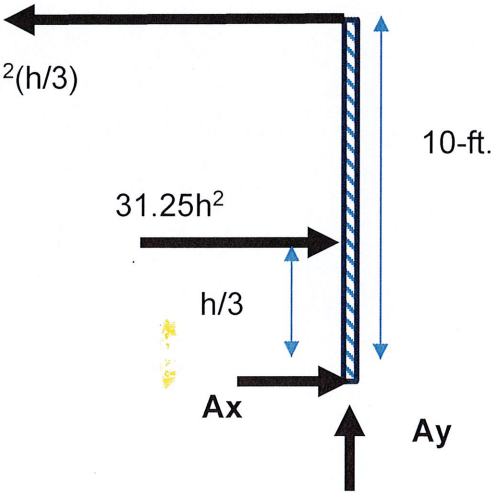
Hydrostatic Force = (Average_Pressure/2)*Area = (1/2)*γ*h*(h*1) = 31.25*h^2

$(\bar{y}_{eq})_{from\ A} = h/3$

Sum of Moments About **A** = 0 = 500*10 – 31.25h^2(h/3)

h^3= (5000*3)/31.25

h= 7.83 '



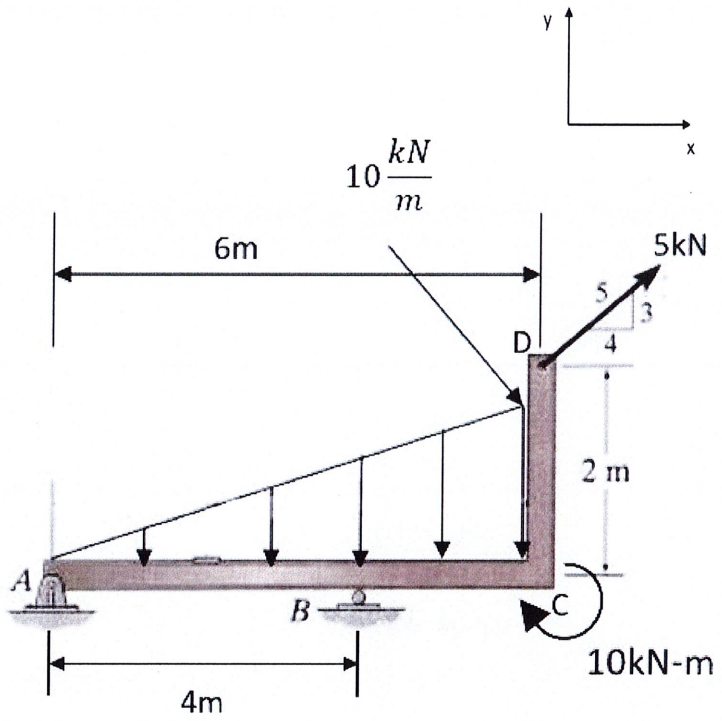
$(\bar{y}_{eq})_{from\ A} = h/3$ (1 pt.)

Equivalent Hydrostatic Force in terms of h	**31.25h^2** (2 pts)
Maximum Height to break cable h =	**7.83 ft.** (3 pts)

PROBLEM 2. (20 points)

Given: The L-shaped bar shown is loaded with a 5kN point force at D, a distributed load from A to C (with a max load per meter of 10kN/m at C), and a 10kN-m couple at C as shown and is held in static equilibrium by a pin support at A and a roller support at B.

Find: a) Determine the equivalent force (F_{eq}) for the distributed load and its distance from A $(\bar{x}_{eq})_{\text{from A}}$. (3 pts)

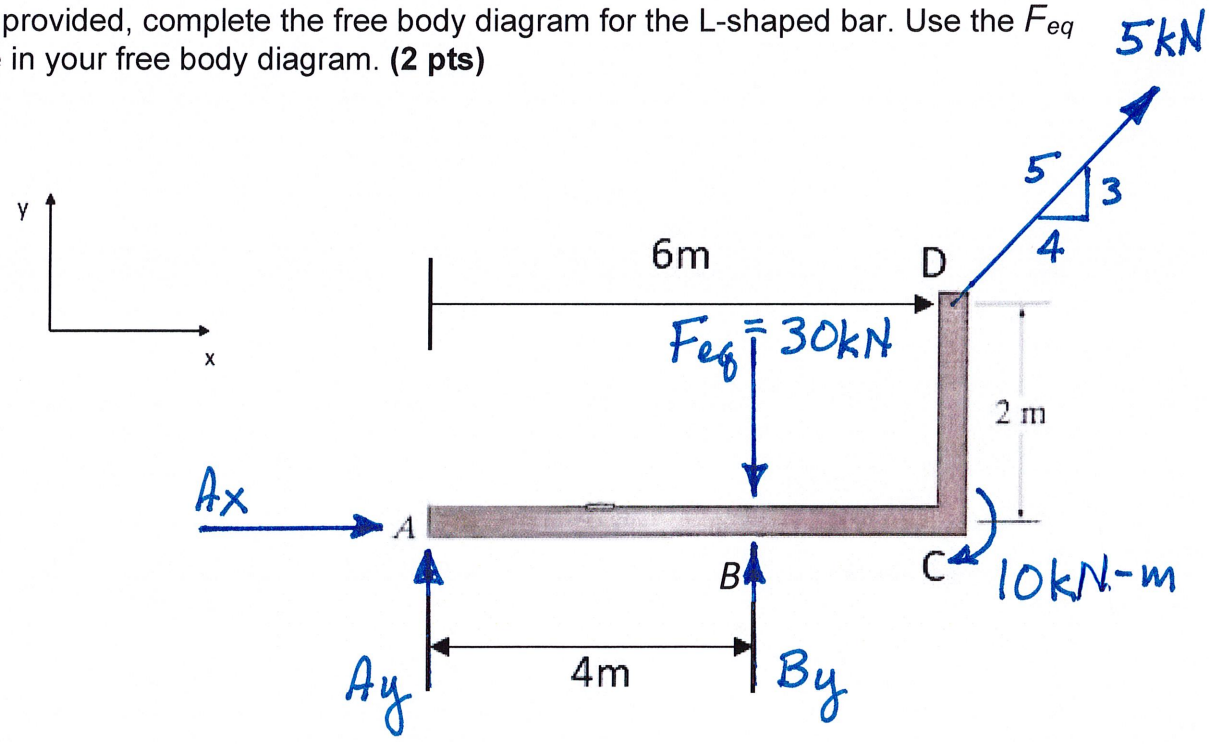


$F_{eq} = \frac{1}{2}(6m)(10\frac{kN}{m}) = \boxed{30kN}$

$(\bar{x}_{eq})_{\text{from A}} = (\frac{2}{3})(6m) = \boxed{4m}$

F_{eq} = 30 kN (2 pts)	$(\bar{x}_{eq})_{\text{from A}}$ = 4 m (1 pt)

b) On the artwork provided, complete the free body diagram for the L-shaped bar. Use the F_{eq} determined above in your free body diagram. **(2 pts)**



c) Clearly write the equilibrium equations and solve for the reactions at the pin support A and the roller support B. Express your solution in vector form. **(12 pts)**

$$\Sigma M_A = 0 = -F_{eq}(4m) + B_y(4m) - \frac{4}{5}(5kN)(2m) + \frac{3}{5}(5kN)(6m)$$

$$-10$$

$$\therefore B_y = +30kN$$

$$\Sigma F_x = 0 = A_x + \frac{4}{5}(5kN) \Rightarrow \therefore A_x = -4kN$$

$$\Sigma F_y = 0 = A_y + B_y - F_{eq} + \frac{3}{5}(5kN)$$

$(+30kN)$ $(+30kN)$

$$\therefore A_y = -3kN$$

$\bar{F}_B =$ $+30$ $\hat{j}$ kN-m (6 pts) $\bar{F}_A =$ -4 $\hat{i}+$ -3 $\hat{j}$ kN (6 pts)

d) If the distributed load were removed from the frame, what qualitative effect would this have on the magnitudes (i.e., neglect any sign changes) of the reactions (no work need be shown)? **(3 pts)**

B_y would:	**increase**	**remain the same**	**decrease**	**(circle one)**	**(1 pt)**
A_x would:	**increase**	**remain the same**	**decrease**	**(circle one)**	**(1 pt)**
A_y would:	**increase**	**remain the same**	**decrease**	**(circle one)**	**(1 pt)**

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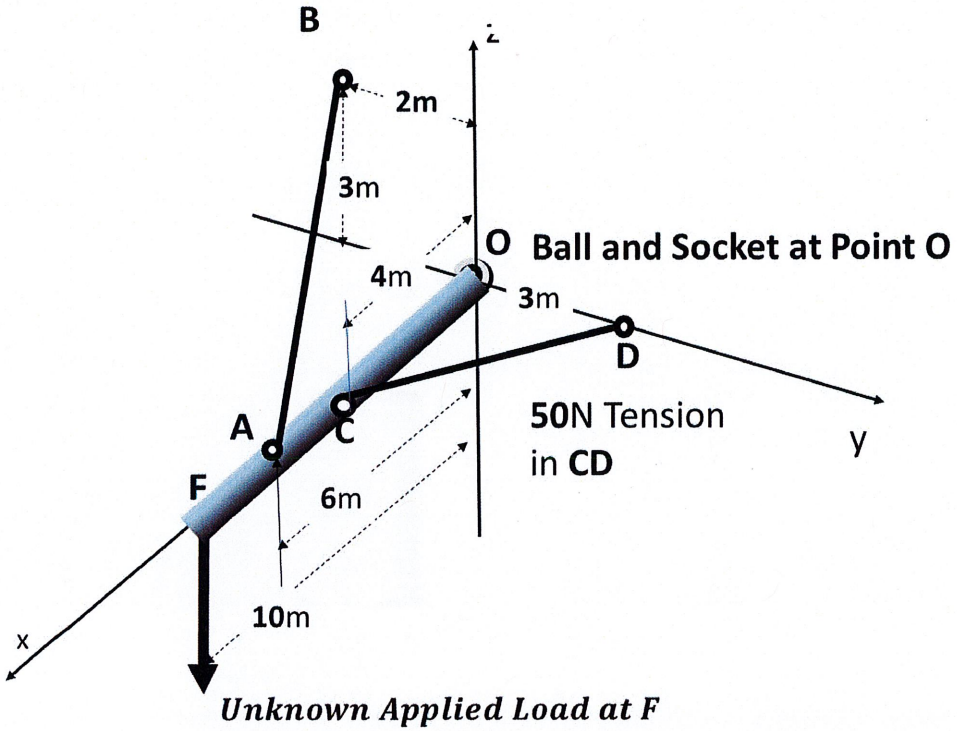
PROBLEM 3. (20 points)

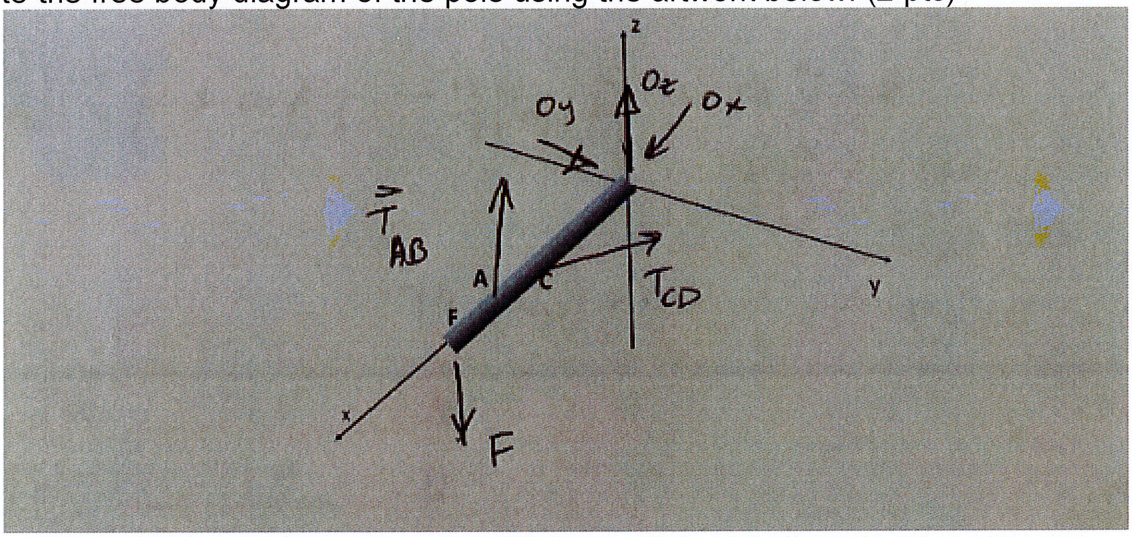
GIVEN:

A pole of negligible mass with an unknown-applied vertical-load at **F** is held in static equilibrium by two cables (**AB** and **CD**) attached to walls at **B** and **D** and a ball-and-socket support at point **O.** The tension in cable **CD** is 50-N.



Point	Location (x, y, z)m
A	(6, 0, 0)m
B	(0,-2, 3)m
C	(4, 0, 0)m
D	(0, 3, 0)m
F	(10, 0, 0)m
O	(0, 0, 0)m

a) Complete the free body diagram of the pole using the artwork below. (2 pts)



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b) Write expressions for tension vectors **AB** and **CD** acting on the pole using their magnitudes (magnitude of **AB** is unknown) and known unit vectors. (4 pts) The applied load is shown as an example, you may express as a **simplified** fraction or use the decimal representation.

$\vec{R}_{AB} = -6\hat{i} - 2\hat{j} + 3\hat{k}$ $\quad \hat{u}_{AB} = -\frac{6}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}$

$\vec{R}_{CD} = -4\hat{i} + 3\hat{j} + 0\hat{k}$ $\quad \hat{u}_{CD} = -\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j} + 0\hat{k}$

$\vec{T}_{AB} = |\vec{T}ab| * [(\ -6/7\ )\hat{i} + (\ -2/7\ )\hat{j} + (\ 3/7\ )\hat{k}]$N (2 pts)

$\vec{T}_{CD} = 50 * [(\ -4/5\ )\hat{i} + (\ 3/5\ )\hat{j} + (\ 0\ )\hat{k}]$N (2 pts)

Applied Load$= |\vec{F}| * [(\ 0\ )\hat{i} + (\ 0\ )\hat{j} + (\ -1\ )\hat{k}]$N (example)

c) Determine the magnitudes of the tension in cables **AB** and the load at **F**. (8 pts)

$\Sigma \vec{M}_O = \vec{0} = \vec{R}_{OA} \times \vec{T}_{AB} + \vec{R}_{OC} \times (-40\hat{i} + 30\hat{j}) + \vec{R}_{OF} \times (-F\hat{k})$

$$|\vec{T}_{AB}| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 0 & 0 \\ -\frac{6}{7} & -\frac{2}{7} & \frac{3}{7} \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 0 \\ -40 & 30 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 0 & 0 \\ 0 & 0 & -F \end{vmatrix}$$

$$-|T_{AB}|\frac{18}{7}\hat{j} + 10F\hat{j} - \frac{12}{7}|T_{AB}|\hat{k} + 120\hat{k} = 0$$

$\frac{12}{7}|\vec{T}_{AB}| = 120$

$T_{AB} = 70$

$70\left(\frac{18}{7}\right) = 10F$

$F = 18$

$|\vec{T}_{AB}| =$ 70 N (4 pts)

$|\vec{F}| =$ 18 N (4 pts)

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d) At point **O**, determine the reactions at **O** and express as a vector. (6 points)

$$\Sigma \vec{F} = \vec{0} = -60\hat{i} - 20\hat{j} + 30\hat{k} - 40\hat{i} + 30\hat{j} - 18\hat{k} + O_x\hat{i} + O_y\hat{j} + O_3\hat{k}$$

$O_x = 100$ N

$O_y = -10$ N

$O_z = -12$ N

$\vec{0} = [(\ 100\ )\hat{i} + (\ -10\ )\hat{j} + (\ -12\ )\hat{k}]$ N. (6 pts)

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