ME 270 – Fall 2024 Final Exam **NAME (Last, First):** SOLUTION

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ______________________________

Instructor's Name and Section: (Circle Your Section)

Sections:

J. Jones, Section 001, MWF 9:30AM-10:20AM
S. Dyke, Section 003, MWF 10:30AM-11:20AM
J. Jones, Section 002, MWF 11:30AM-12:20PM
F. Semperlotti, Section 005, MWF 12:30PM-1:20PM
F. Zhao, Section 008, MWF 1:30PM-2:20PM
F. Semperlotti, Section 009, MWF 2:30PM-3:20PM
A. Arrieta, Section 010, MWF 3:30PM-4:20PM
M. Murphy, Section 007, TR 9:00AM-10:15AM
J. Jones, Section Y01, Distance Learning

Please review and sign the following statement:

Purdue Honor Pledge – "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

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INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 25 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

- The allowable exam time for Final Exam is 120 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the back side of your exam paper.

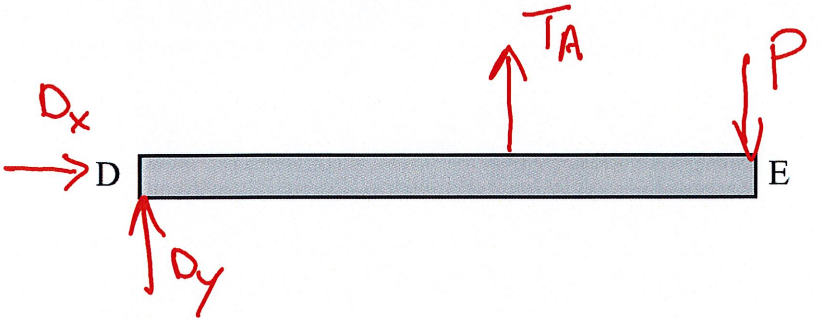
If the solution does not follow a logical thought process, it will be assumed in error.

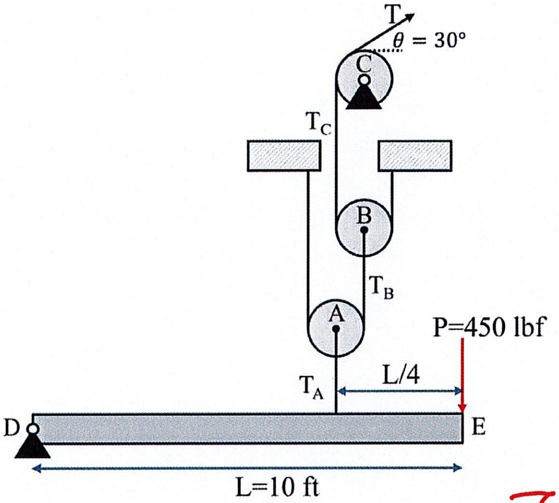
When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of the cover page. Also, be sure to identify the page numbers for each problem before final submission on Gradescope. Do not include the cover page or the equation sheet with any of the problems.

PROBLEM 1 (25 points)

1A. Consider the system in figure made by a rigid bar hinged at point D and connected to a system of three ideal (i.e., frictionless) pulleys A, B, and C. If the bar is loaded at point E by a load P=450 lbf and is in static equilibrium, **find** the tension T_A in the cable at equilibrium and the tension T in the cable at equilibrium. **(6 pts)**

FBD of Bar DE (2 pts)



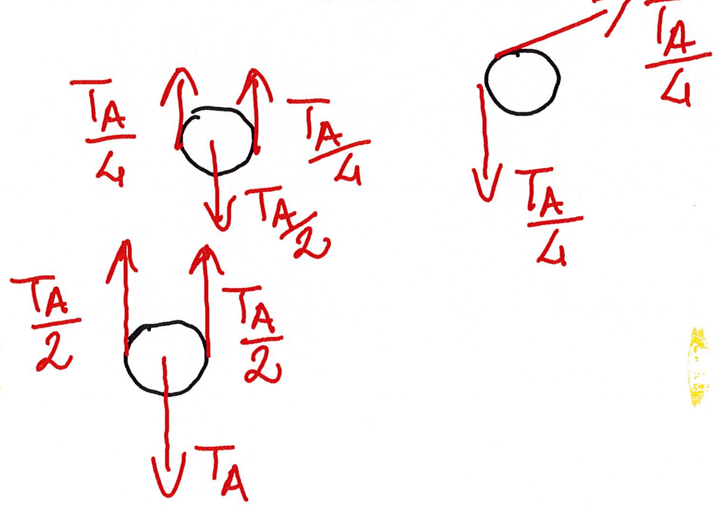


$$\Sigma M_D = 0 \qquad T_A \cdot \left(\frac{3L}{4}\right) - PL = 0$$

$$T_A = \frac{4P}{3} = 600 \text{ lbs}$$

PULLEY:

$$T = \frac{T_A}{4} = 150 \text{ lbs}$$



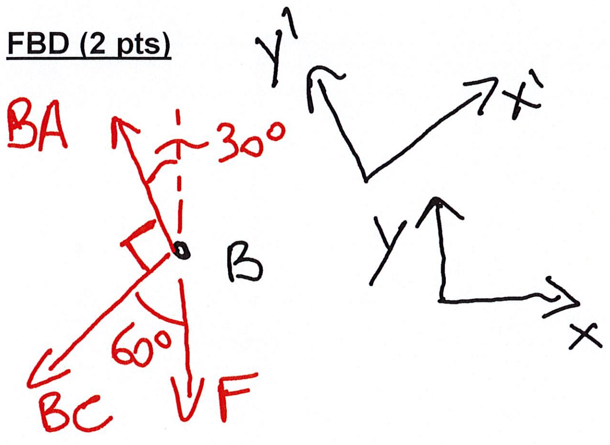
T_A = 600 lbf (2 pts) T = 150 lbf (2 pts)

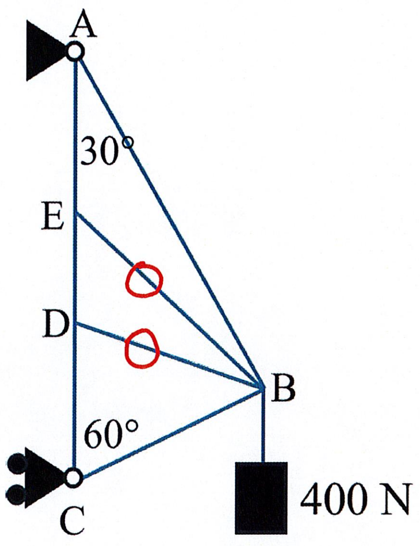
1B. (6 pts) The truss in figure is loaded by a crate applying a force of 400 N.

a- **Identify** all the zero-force members. Circle your choices in the box below. No calculation needs to be shown.

b- Using the method of joints, **find** the load in element BA. Is the element loaded in tension or compression (Circle your choice)? Include a FBD in your work.

FBD (2 pts)





IN x-y frame:

x: $-AB\sin 30° - BC\sin 60° = 0$

y: $AB\cos 30° - F - BC\cos 60° = 0$

$BC = -AB\frac{\sin 30°}{\sin 60°}$; $AB = \frac{1}{\cos 30°}\left[F + BC\cos 60°\right] =$

$= \frac{F}{\cos 30°} + AB\frac{\tan 30°}{\tan 60°}$

$AB = \frac{F}{\cos 30°}\cdot\left(1+\frac{\tan 30°}{\tan 60°}\right)^{-1} = 346.41$ N

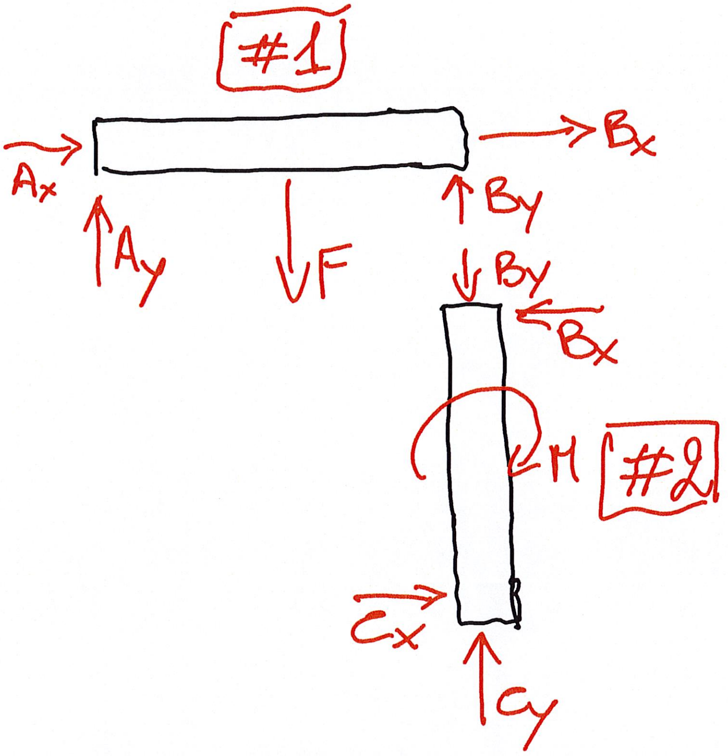
In x'-y': x': $-BC - F\cos 60° = 0$ $BC = -F\cos 60° = -200$N

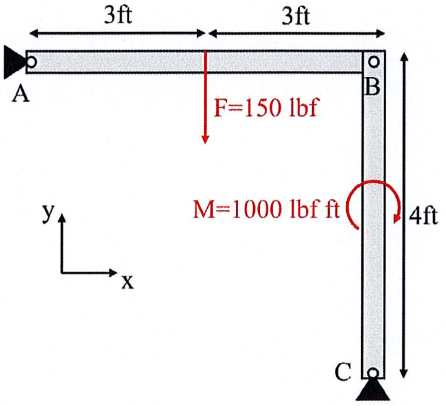
y': $BA - F\sin 60° = 0$ $BA = F\sin 60° = 346.41$N

Zero force members	AE	ED	DC	[✓ BE]	[✓ BD]	BA	BC	(2 pts)
BA = 346.41 N	[✓ Tension]	Compression	Zero	(Circle One)				(2 pts)

1C. Given the frame in figure, subject to a force F=150 lbf and a couple M= 1000 ft-lbf, **find** the value of the internal force at joint B on Member AB. Express the result in vector form. If the pin is rated for a maximum force of 150 lbf, will the pin break? (**HINT:** You do not need to solve the overall frame to determine the forces at joint B.) **(6 pts)**

FBDs (2 pt)





#1: $\Sigma M_A = 0 \quad -F \cdot 3 + B_y \cdot 6 = 0 \quad \boxed{B_y = \frac{F}{2} = 75 \text{ lbs}}$

#2: $\Sigma M_C = 0 \quad B_x \cdot 4 - M = 0 \quad \boxed{B_x = \frac{M}{4} = 250 \text{ lbs}}$

$B = \sqrt{B_x^2 + B_y^2} = 261 \text{ lbs}$

261 lbs > F_{MAX} = 150 lbs

PIN BREAKS

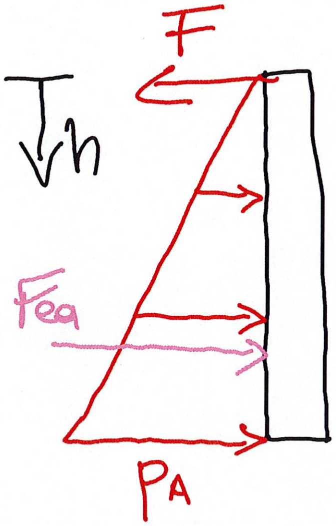
$B = (75)\hat{\imath} + (250)\hat{\jmath}$ lbf	(2 pts)
Does the pin break? [✓ Yes] No Cannot Tell (Circle One)	(2 pts)

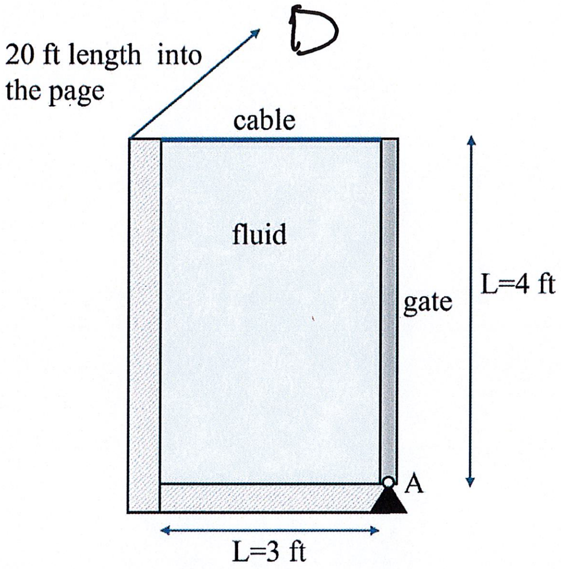
on AB

1D. Consider the container in figure closed by a vertical gate hinged at point A and restrained at the top by 10 equally spaced cables distributed along the 20ft length of the container. Assume the container is filled to the top with fresh water having specific density $\rho g = 62.4\ lbs/ft^3$. **(7 pts)**

a) Assuming equilibrium, **find**

b) Determine water pressure p_A at the bottom of the container at A. **(2 pts)**
c) Determine the Feq on the gate. **(2 pts)**
d) Determine the force **F** in a single cable when the container is filled. Include a FBD of the gate. **(3 pts)**





$$P_A = (\rho g)_{WATER} \cdot h = 62.4 \frac{lbs}{ft^3} \cdot 4\ ft = 249.6 \frac{lbs}{ft^2}$$

$$F_{eq} = \frac{1}{2} P_A \cdot L \cdot D = 9984\ lbs$$

$$\Sigma M_A = 0 \qquad F \cdot 4 - F_{eq} \cdot \frac{L}{3} = 0 \qquad F = \frac{F_{eq}}{3}$$

$$= 3328\ lbs$$

p_A = 249.6 ~~psi~~ $\frac{lbs}{ft^2}$ (2 pts) Feq = 9984 lbf (2 pts) F = 3328 lbf (2 pts)

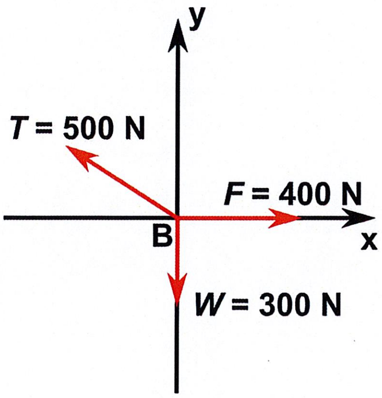
PROBLEM 2 (25 points)

2A. Consider a beam AB with a pin support on the wall at point A, and the other end B is held up by cable BC. The cable BC is attached to the wall at point C. As such, the structure acts like a small truss as both the cable and beam AB can be considered 2-force members. A weight $W = 300\ \mathrm{N}$ is hanging from point B. The beam AB has a hollow squared cross-section with outer dimensions of $a = 7\ \mathrm{mm}$ and thickness of $t = 2\ \mathrm{mm}$ as shown in the figure below. Ignore the weight of beam AB and cable BC. **(6 pts)**

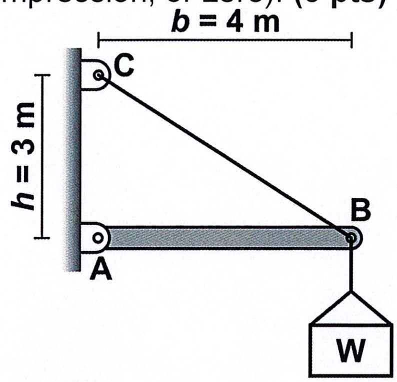
Find:

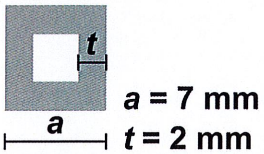
a) Sketch FBD at joint B. **(1 pt)**
b) Determine the tension of cable BC T_{BC} and the internal force of beam AB F_{AB}. **(2 pt)**
c) Determine the stress acting on beam AB and its sense (tension, compression, or zero). **(3 pts)**

FBD of point B:



(1 pt)





Cross-Section of Beam AB

b) According to the FBD, consider y direction:

$$\sum F_y = 0:\ -W + T_{\mathrm{BC}} \times \frac{3}{5} = 0 \Rightarrow \boldsymbol{T_{\mathrm{BC}}} = \frac{\mathbf{5}}{\mathbf{3}}\boldsymbol{W} = \mathbf{500\ N},$$

$$\sum F_x = 0: F_{\mathrm{AB}} - T_{\mathrm{BC}} \times \frac{4}{5} = 0 \Rightarrow \boldsymbol{F_{\mathrm{AB}}} = \frac{\mathbf{4}}{\mathbf{5}}\boldsymbol{T_{\mathrm{BC}}} = \mathbf{400\ N}.$$

c) The area of the cross-section is given by:

$$\boldsymbol{A} = a^2 - (a - 2t)^2 = 49 - (7 - 2 \times 2)^2 = \mathbf{40\ mm^2} = \mathbf{4 \times 10^{-5}\ m^2}.$$

Therefore, the stress at beam AB is:

$$\boldsymbol{\sigma_{\mathrm{AB}}} = \frac{\boldsymbol{F_{\mathrm{ab}}}}{\boldsymbol{A}} = \mathbf{10\ MPa}.$$

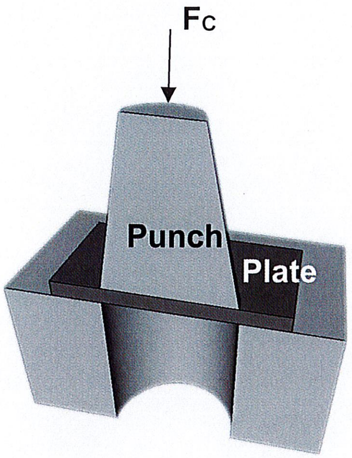
AB is under **compression**.

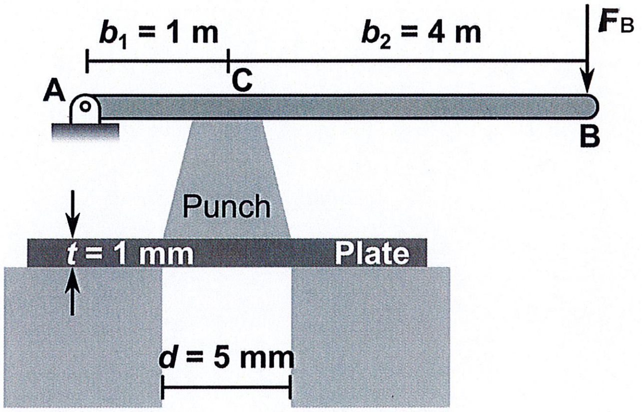
T_{BC} = 500 N	**(1 pt)**
F_{AB} = 400 N	**(2 pts)**
σ_{AB} = 10 MPa	**(2 pts)**
Circle the sense of the stress of AB: Tension, [✓ Compression], Zero.	**(1 pt)**

2B. Consider a manual punch press is used to punch a circular hole in an aluminum plate with a thickness $t = 1\ \mathrm{mm}$. The left figure below shows half of the punch and the plate (lever not shown), and the right figure below shows the cross-section of the manual punch. The diameter of the hole is $d = 5\ \mathrm{mm}$. Force F is applied on point B of a lever AB to push the punch press. The length of segment AC is $b_1 = 1\ \mathrm{m}$ and the length of segment CB is $b_2 = 4\ \mathrm{m}$. Assume C is the point that the force is applied to the punch press and the weight of the lever is negligible. Given the shear strength of aluminum is $\tau = 270\ \mathrm{MPa}$. **(6 pts)**

Find: a) the magnitude of the minimum force at C to punch the hole on the plate (F_C). **(3 pts)**

b) the minimum force on the end of the lever at B to punch the hole on the plate (F_B). Include a FBD. **(3pts)**



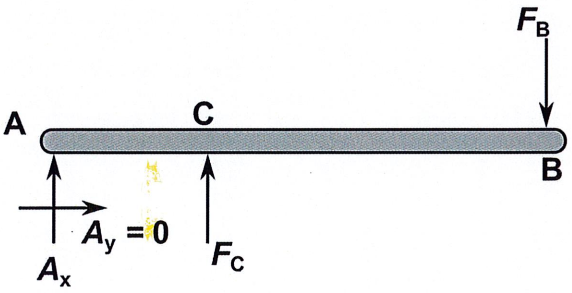


$$C = \pi d = 5\pi\ \mathrm{mm}$$

$$A = Ct = 5\pi\ \mathrm{mm}^2$$

$$F_\mathrm{p} = \boldsymbol{F_\mathrm{B}} = \tau A = \mathbf{1350\pi\ N}$$

$$\boldsymbol{F_\mathrm{B}} = \frac{b_1}{b_1 + b_2} F = 270\pi\ \mathrm{N} = \mathbf{848.23\ N}$$

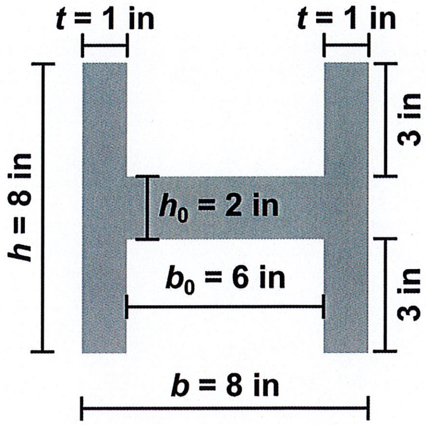


Fc = 4241.1 N (3 pts) FB = 848.23 N (3 pts)

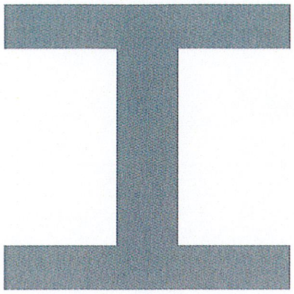
2C. (6 pts) Consider an H-beam as shown in the left figure below. The width b and the height h are both equal to 8 in, with a width of the flanges t equals to 1 in and the thickness of the web h_0 equals to 2 in as shown in the figure below.

a) Find the second moment of inertia of the H-beam I_x. **(4 pts)**

b) If we rotate the H-beam for 90° to become an I-beam as shown in the right figure below, qualitatively, would I_x of the I-beam be greater, smaller, or the same compared with the H-beam. **(2 pts)**



H-Beam



I-Beam

a)

For the left and right flanges:

$$I_\mathrm{xf} = \frac{1}{12}th^3 = \frac{1}{12} \times 1 \times 8^3 = \frac{128}{3}\ \mathrm{in}^4 = 42.667\ \mathrm{in}^4.$$

For the web:

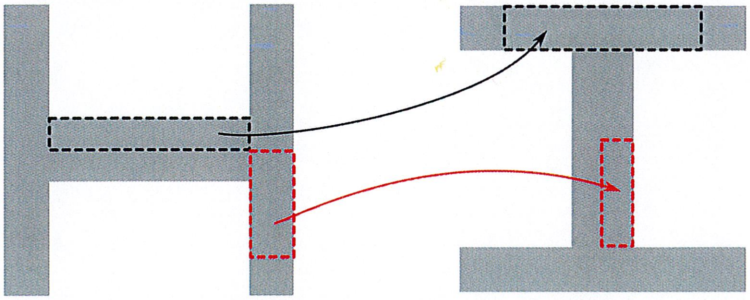
$$I_\mathrm{xw} = \frac{1}{12}b_0h_0^3 = \frac{1}{12} \times 6 \times 2^3 = 4\ \mathrm{in}^4.$$

Therefore, the second moment area of inertia of the H-beam is:

$$\boldsymbol{I_\mathrm{x}} = 2I_\mathrm{xf} + I_\mathrm{xw} = \frac{268}{3}\ \mathrm{in}^4 = \mathbf{89.333\ in^4}.$$

b)

Greater because more area is far from its neutral axis.

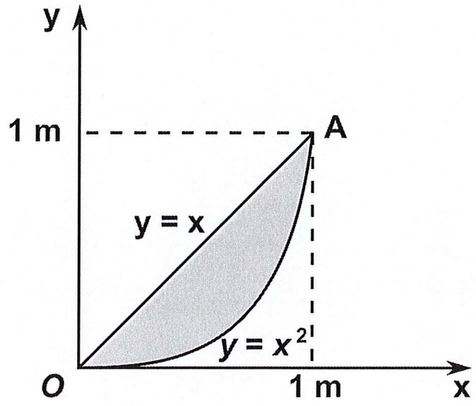


I_x = 89.333 in^4	**(4 pt)**
I_x will be ([✓ greater], smaller, the same) after rotation. (Circle the correct answer.)	**(2 pt)**

2D. Given a shaded region enclosed by a parabola and a line segment AO as shown in the figure below. The parabola is given by $y = x^2$ and the line is given by $y = x$. Point A has a coordinate of $(1, 1)$ m. Assume I_x is the second area moment of inertia about the x-axis, I_y is the second area moment of inertia about the y-axis and I_{yc} is the second moment of area about the y-centroid. **(7 pts)**

Find:

a) The second area moment of inertia about y-axis I_y. **(3 pts)**
b) Based on the shaded area, would you expect I_x to be greater than, equal to or smaller than I_y? **(2 pts)**
c) Based on the shaded area, would you expect I_{yc} to be qualitatively greater, equal to, or less than I_y? **(2 pts)**



a)

$$I_y = \int_0^1 x^2(x - x^2)\, dA = \left[\frac{1}{4}x^4 - \frac{1}{5}x^5\right]_0^1 = \frac{\mathbf{1}}{\mathbf{20}}\ \mathbf{mm^4} = \mathbf{0.05\ mm^4}.$$

b)

$$\boldsymbol{I_x < I_y}$$

c)

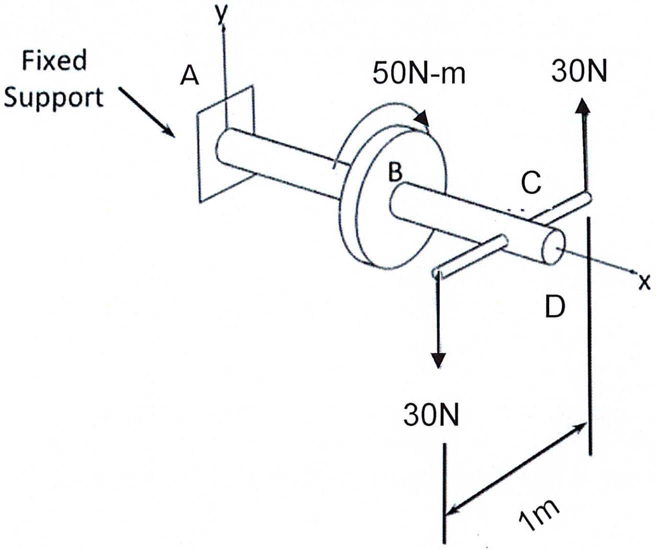
$$\boldsymbol{I_{yc} < I_y}$$

$I_y =$	0.05		m^4	**(3 pt)**
$I_x > I_y$	$I_x = I_y$	[✓ $I_x < I_y$]	**(Circle One)**	**(2 pt)**
$I_{yc} > I_y$	$I_{yc} = I_y$	[✓ $I_{yc} < I_y$]	**(Circle One)**	**(2 pt)**

PROBLEM 3 (25 points)

Given: A solid circular shaft ABCD has a coupler disk B with a 50N-m couple applied as shown and a crossbar at point C with two 30N forces acting perpendicular to the crossbar in opposite directions and separated by 1m as shown. The shaft is held in static equilibrium by a fixed support at A with its positive direction x defined from origin A to D. The radius of the shaft is $r = 50\ \text{mm}$.

a) Find the magnitude of the torques T_{AB}, T_{BC}, and T_{CD} at segment AB, BC, and CD **(6 pt)**.

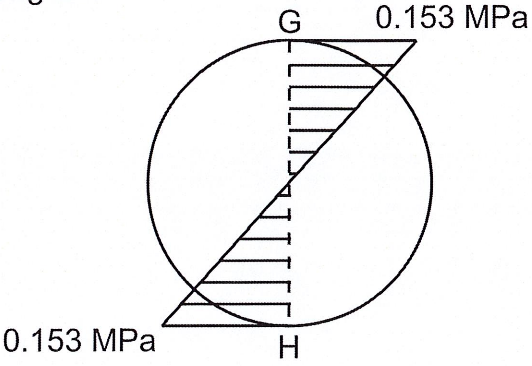


$$T_{\text{DC}} = 0$$
$$T_{\text{BC}} = -Fb = -30\ \text{N}\cdot\text{m}$$
$$T_{\text{AB}} = -(T_{\text{BC}} + M) = -(30 - 50) = 20\ \text{N}\cdot\text{m}$$

$|T_{AB}|$ = 20 N-m **(2 pts)** $|T_{BC}|$ = 30 N-m **(2 pts)** $|T_{CD}|$ = 0 N-m **(2 pts)**

b) Find the maximum shear stress on shaft AD **(4 pt)** and sketch the shear stress profile of the cross-section where the maximum shear stress is observed in the artwork provided below **(2 pt)**. If the radius of the solid shaft increases, what would happen with the maximum shear stress **(2 pt)**?

Sketch shear force profile along GH:



$$J = \frac{1}{2}\pi r^4 = 9.81 \times 10^{-6}\ \mathrm{m}^4$$

$$\boldsymbol{\tau}_{\mathbf{max}} = \frac{T_{\max} r}{J} = \frac{30 \times 0.05}{9.81 \times 10^{-6}} = \mathbf{153\ kPa}$$

$|\tau|_{max}$ = **153** **kPa (4 pts) If radius increases, $\boldsymbol{\tau}_{max}$:** increases, [✓ decreases], remains the same (2 pts)

c) If the shaft is made of a hollow circular tube with an outer radius of $r_o = 50\ \mathrm{mm}$. Consider the maximum shear stress at a point in **segment BC** is $\tau_{\max} = 1\ \mathrm{MPa}$, find the thickness (**t**) of the hollow shaft. **(5 pts)**.

$$\tau_{\max} = \frac{2T_{\mathrm{BC}} r_o}{\pi(r_o^4 - r_i^4)} \Rightarrow r_i = \sqrt[4]{r_o^4 - \frac{2T_{\mathrm{BC}} r_o}{\tau_{\max}\pi}} = 48\ \mathrm{mm}$$

$$\boldsymbol{t} = r_o - r_i = \mathbf{2\ mm}$$

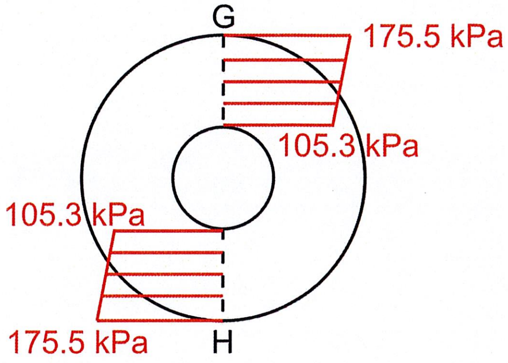
t = r

$t = r_o - r_i$ = 2 mm **(5 pts)**

d) In **section BC** of the shaft if the outer radius was r_o = 50mm and the inner radius was r_i = 30mm, determine the minimum shear stress **(4 pts)** and sketch the shear stress profile of the cross-section in the artwork provided below **(2 pts)**.

(2pts)

Sketch shear force profile along GH:



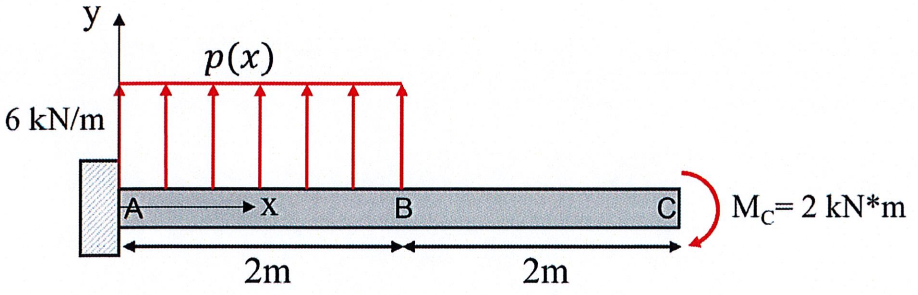
$$\tau_{\text{min}} = \frac{2T_{\text{BC}}r_{\text{i}}}{\pi(r_{\text{o}}^4 - r_{\text{i}}^4)} = \frac{2 \times 30 \times 0.03}{\pi \times (0.05^4 - 0.03^4)} = \mathbf{105.3\ kPa}$$

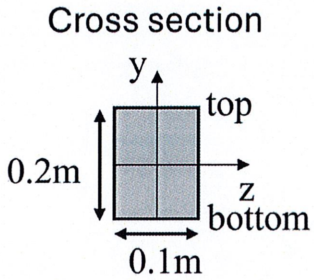
$$\tau_{\text{max}} = \frac{2T_{\text{BC}}r_{\text{o}}}{\pi(r_{\text{o}}^4 - r_{\text{i}}^4)} = \frac{2 \times 30 \times 0.05}{\pi \times (0.05^4 - 0.03^4)} = 175.5\ \text{kPa}$$

$\lvert\tau\rvert_{min}$ = ______105.3______ kPa	**(4 pts)**

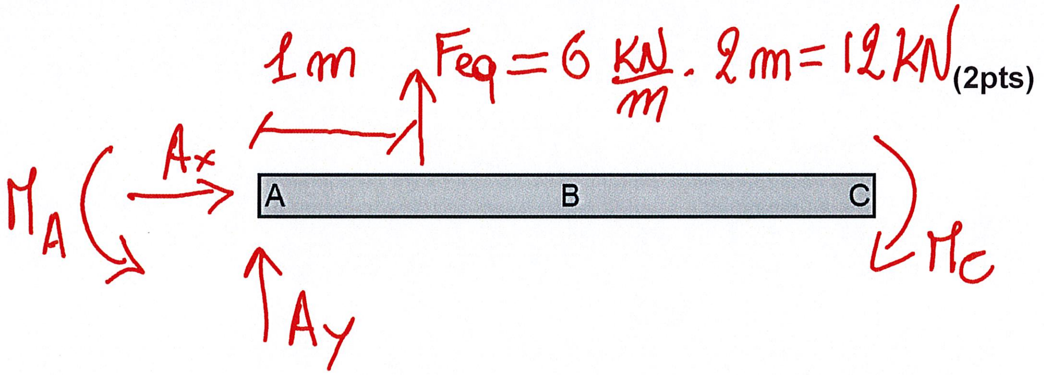
PROBLEM 4 (25 points)

Consider the cantilever beam depicted in the figure below. The beam is fixed at point A and subject to a uniform distributed load p(x) and a couple M_C at C, as shown in figure. The beam has a rectangular cross section and is made of steel with a Young's modulus $E = 200GPa$ and ultimate stress $\sigma_U = 110MPa$.





a) **Draw** the overall free body diagram on the artwork provided below **(2 pts)**. Use the single force evaluated to the distributed uniform load. **Determine** the reactions at the fixed end A and write them in the box below. **(7 pts)**



$$\Sigma M_A = 0 \qquad M_A - M_c + F_{eq} \cdot 1 = 0$$

$$\boxed{M_A = -10 \text{ KN} \cdot m}$$

$$\Sigma F_x = 0 \qquad A_x = 0$$

$$\Sigma F_y = 0 \qquad A_y + F_{eq} = 0 \qquad \boxed{A_y = -12 \text{ KN}}$$

$A = (\underline{\quad 0 \quad})\hat{\imath} + (\underline{\quad 12 \quad})\hat{\jmath}$ kN **(3pts)**

$M_A = (\underline{\ -10\ })\hat{k}$ kN-m **(2 pts)**

b) **Find** the expressions describing the distribution of the shear force V(x) and the bending moment M(x) as a function of the spatial coordinate x. Write your results for the segments AB and BC in the box below. **(8 pts)**

AB

$$V(x) = V(0) + \int_0^x p(\bar{x})\,d\bar{x} = A_y + 6x$$

$$M(x) = M(0) + \int_0^x V(\bar{x})\,d\bar{x} = -M_A + (A_y x + 3x^2)$$

$$= 10 - 12x + 3x^2$$

$M(0) = 10$ KN·m

$M(2) = -2$ KN·m

BC

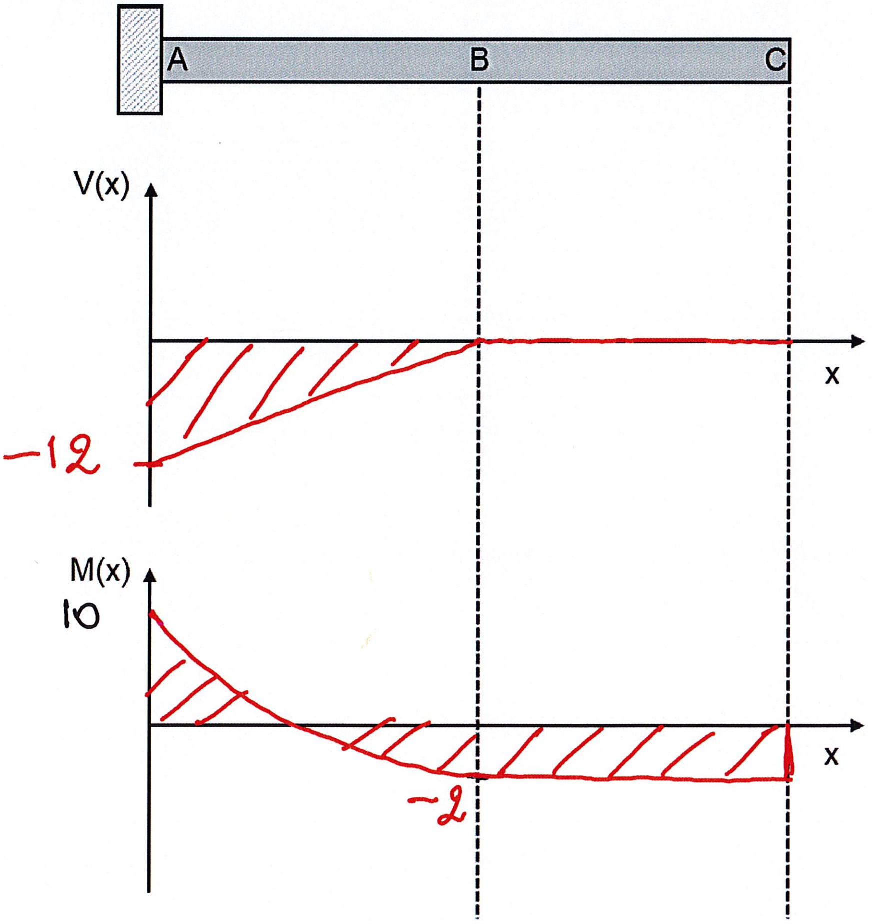
$$V(x) = V(2) + \int_0^x \cancelto{0}{p(\bar{x})}\,d\bar{x} = 0$$

$$M(x) = M(2) + \int_0^x V(\bar{x})\,d\bar{x} =$$

$$= -2$$

Segment AB	V(x) = $-12 + 6x$	kN	M(x)= $10 - 12x + 3x^2$	kN-m	(4 pts)
Segment BC	V(x) = 0	kN	M(x)= -2	kN-m	(4 pts)

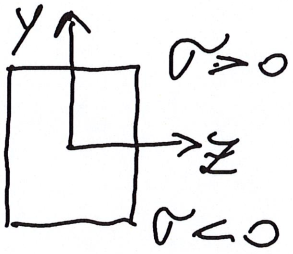
c) **Draw** the diagram V(x) and M(x) on the diagram provided below **(4 pts)**



d) Indicate over what segments or points pure bending conditions occur (**Circle your choice** in the box below)? For the region(s) experiencing pure bending, **find** the value of the maximum compressive normal stress and the location on the cross section where it occurs. **(6 pts)**

$$\sigma_{BOTTOM} = -\frac{My}{I_z} = -\frac{2\cdot10^3\ N\cdot 0.1\ m}{6.66\cdot10^{-5}\ m^4} = -3\cdot10^6\ Pa = -3MPa$$

$$I_z = \frac{0.1\cdot 0.2^3}{12} = 6.66\cdot10^{-5}\ m^4$$



Pure bending	Segment AB [✓ Segment BC] Segment AC Point A (Circle all that apply)	(2 pts)
Max compressive stress	$\sigma =$ 3 **MPa**	(2 pts)
Location of max compressive stress:	Top [✓ Bottom] Center (Circle One)	(2pts)