ME 270 Exam 1 – Spring 2025 **NAME (Last, First):** SOLUTION **(Please Print)**

PUID #: ____________________ **(Please Print)**

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ______________________________

Instructor's Name and Section: (Circle Your Section)

Sections:

J. Jones, Section 001, MWF 9:30AM-10:20AM (WL)
S. Dyke, Section 013, MWF 10:30AM-11:20AM (WL)
J. Jones, Section 003, MWF 11:30AM-12:20PM (WL)
L. Krest, Section 005, MWF 1:30PM-2:20PM (WL)
J. Jones, Section 004, Distance Learning (WL)
A. McDonald, Section 015, MWF 3:30PM-4:20PM (Indy)

Please review and sign the following statement:

Purdue Honor Pledge – "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

Signature: ______________________________

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

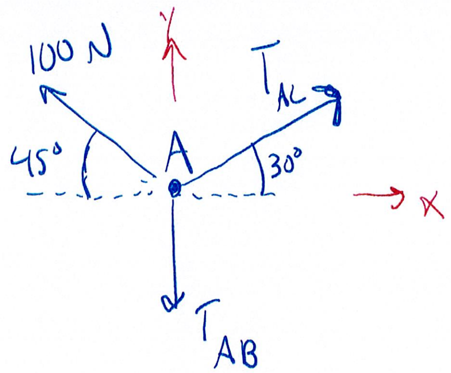
- The allowable exam time for Exam 1 is 90 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the back side of your exam paper.

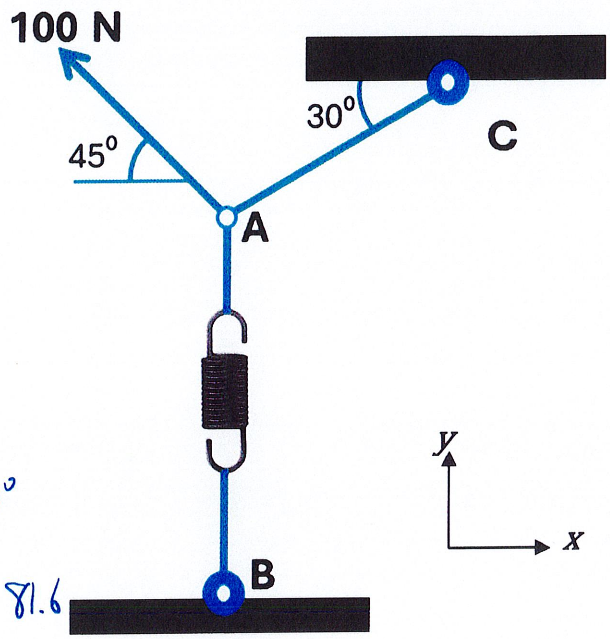
If the solution does not follow a logical thought process, it will be assumed in error.

When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of the cover page. Also, be sure to identify the page numbers for each problem before final submission on Gradescope. Do not include the cover page or the equation sheet with any of the problems.

PROBLEM 1 (20 points)

1A. A cable system in static equilibrium is shown Determine the magnitudes of tension in cable AC and spring AB. Include a free body diagram and write clear equation of static equilibrium.
(Hint: treat the spring like a cable) **(5 pts)**





$$\Sigma F_x = 0 = -100\cos 45^\circ + T_{AC}\cos 30^\circ$$

$$T_{AC} = \frac{100\cos 45^\circ}{\cos 30^\circ} = \frac{100\sqrt{6}}{3} = 81.6$$

$$\Sigma F_y = 0 = 100\sin 45^\circ + T_{AC}\sin 30^\circ - T_{AB}$$

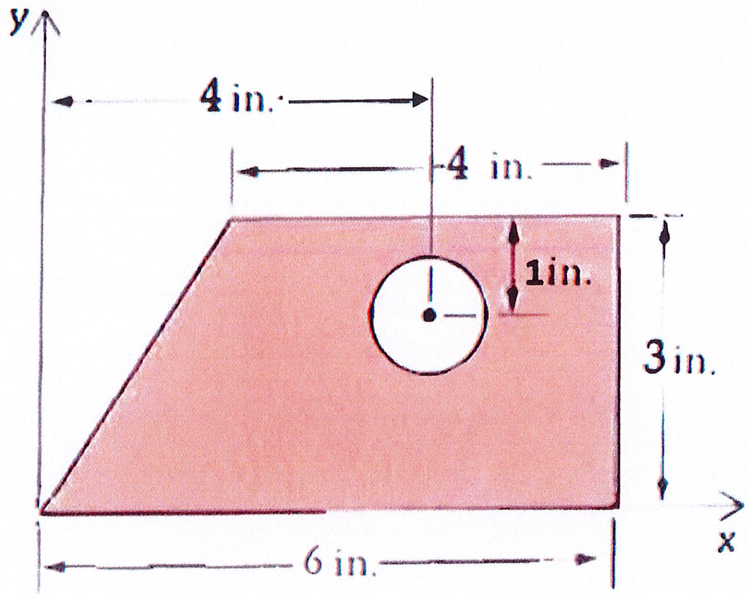
$$T_{AB} = 100\sin 45^\circ + 81.6\sin 30^\circ = 111.5\text{ N}$$

FBD (1 pt)

$\lvert\vec{T}_{AC}\rvert =$ 81.6 N	**(2 pts)**
$\lvert\vec{T}_{AB}\rvert =$ 112 N	**(2 pts)**

1B. Using the method of composite parts, find the area (A) and the x-centroid (x_C) of the shaded area in the figure below with respect to the coordinate axes provided. Please show your work to receive credit. The circular hole has a diameter of 1 in. If the triangular gap next to the y-axis was filled in (making the shape rectangular with a hole in it), qualitatively what impact would this have on x_C and y_C. (No calculations are required for determining this qualitative impact). **(5 pts)**

	Area (in^2)	$\bar{x}$ (in)
Triangle	$3\ in^2$	$4/3$ in
Rectangle	$12\ in^2$	4 in
Circle	$-\pi/4\ in^2$	4 in
A_{TOT} =	$14.22\ in^2$	



$$A_{TOT}\, x_c = A_1 \bar{x}_1 + A_2 \bar{x}_2 + A_3 \bar{x}_3$$

$$(14.22 in^2)\, x_c = (3 in^2)(1.333 in) + (12 in^2)(4 in) + \left(-\frac{\pi in^2}{4}\right)(4 in)$$

$$(14.22 in^2)\, x_c = 48.86$$

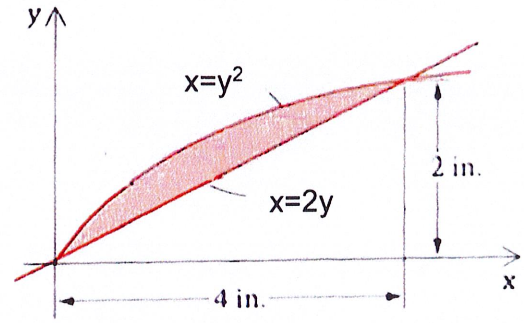
$$\boxed{\therefore\ x_c = 3.44\ \text{in}}$$

A = 14.22 in^2 (1 pt) x_C = 3.44 in (2 pts)

x_C = Increase Stay the Same Decrease (Circle One) (1 pt)

y_C = Increase Stay the Same Decrease (Circle One) (1 pt)

1C. Using the method of integration, determine the area (A) and the x-centroid (x_C) of the shaded area with respect to the coordinate axes provided. Please show your work to receive credit. Qualitatively, would you expect for y_C to be larger, smaller or equal to x_C? (No calculations are required for determining this qualitative impact). **(5 pts)**



$$A = \int_{y_1=0}^{y_2=2} \int_{x_1=y^2}^{x_2=2y} dx\,dy$$

$$A = \int_0^2 [x]_{x_1=y^2}^{x_2=2y} dy = \int_0^2 (2y - y^2)\,dy$$

$$A = \left[\frac{2y^2}{2} - \frac{y^3}{3}\right]_{y_1=0}^{y_2=2} = 4 - \frac{8}{3} = \boxed{1.333\text{ in}^2}$$

$$A x_c = \int_{y_1=0}^{y_2=2} \int_{x_1=y^2}^{x_2=2y} x\,dx\,dy = \int_0^2 \left[\frac{x^2}{2}\right]_{x_1=y^2}^{x_2=2y} dy$$

$$A x_c = \int_0^2 \left(\frac{4y^2}{2} - \frac{y^4}{2}\right) dy = \left[\frac{2y^3}{3} - \frac{y^5}{10}\right]_{y_1=0}^{y_2=2}$$

$$(1.333\text{ in}) x_c = \frac{16}{3} - \frac{32}{10} = 2.1333$$

$$\boxed{\therefore x_c = 1.6\text{ in}}$$

A = 1.333 in² (2 pts) xc = 1.6 in (2 pts)

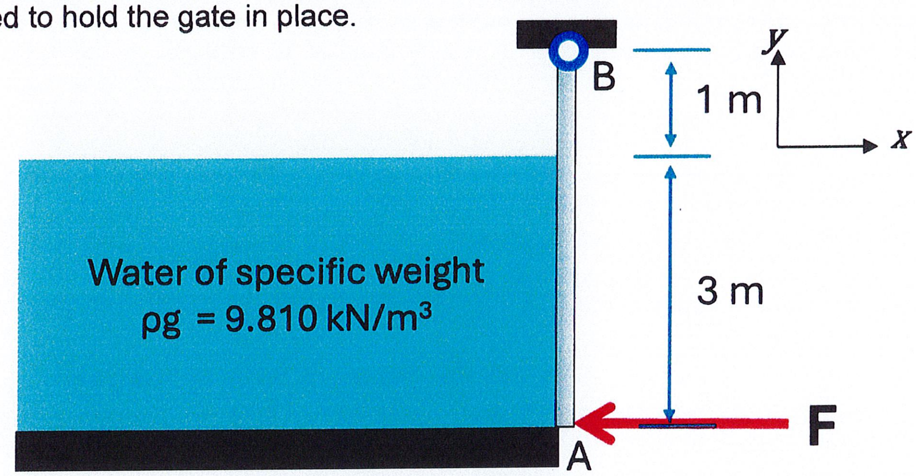
y_C Trend: $y_C > x_C$ $y_C < x_C$ $y_C = x_C$ (Circle One) (1 pt)

1D. A 2-meter-wide gate hangs from a pin joint at B, while force F applied at point A holds the gate in place. Water at 3 m depth is held on one side of the gate. The specific weight of the water is ρg = 9.81 kN/m^3 **(5 pts)**

Please Determine:

i. The equivalent hydrostatic force (F_{eq}) applied to the 2-meter-wide gate.

ii. The force F required to hold the gate in place.



$$P_A = \rho g h = 9.81\ \frac{kN}{m^3}(3m) = 29.43\ \frac{kN}{m^2}$$

$$F_{eq} = \frac{1}{2}P_A(3m)(2m) = 88.29\ kN$$

$$\Sigma M_B = F_{eq}(3m) - F(4m)$$

$$F = F_{eq}\frac{3}{4} = 66.22\ N$$

F_{eq} = 88.3 kN		**(2 pts)**
F = 66.2 kN		**(3 pts)**

PROBLEM 2. (20 points)

Given: Bar ABCDE is loaded with a point force at B, a distributed load on the T-bar, and a point moment at D and is held in static equilibrium by a pin support at A and a roller support at E. The distributed load varies linearly from zero on each end to a maximum value of 8ft-lbs in the middle of the 2 ft long T-bar.

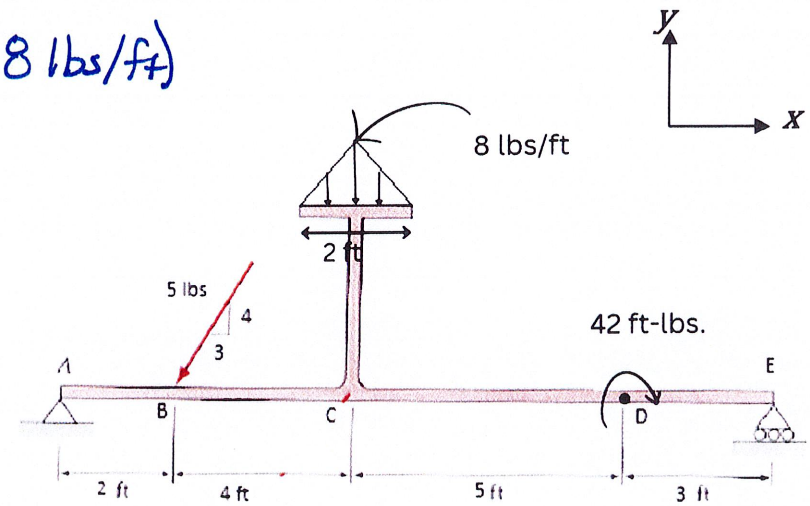
Find: a) Determine the equivalent force (F_{eq}) for the distributed load and its x-distance from A ($(\bar{x}_{eq})_{\text{from A}}$). **(3 pts)**

$F_{eq} = \frac{1}{2} b h = \frac{1}{2}(2ft)(8\ lbs/ft)$

$F_{eq} = 8\ lbs.$

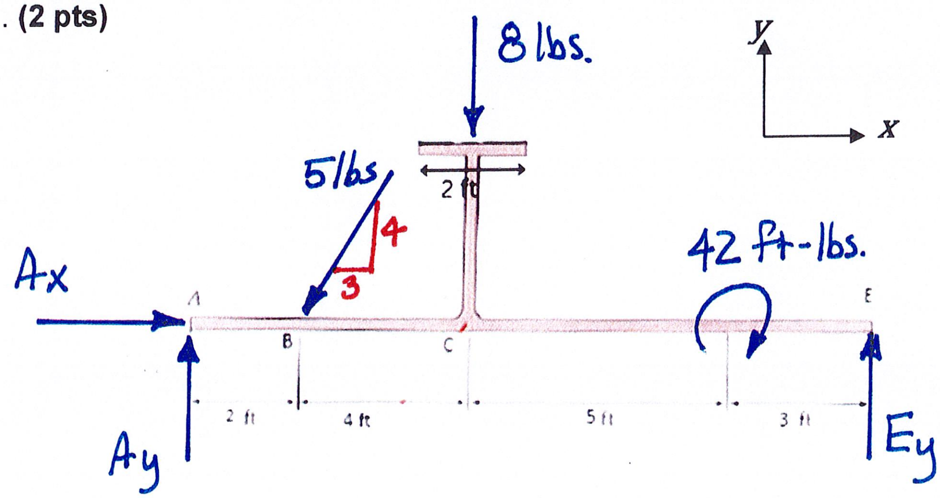
$(x_{eq})_{\text{from A}} = 6\ ft.$

(based on symmetry)



F_{eq} = 8 lbs (2 pts) $(\bar{x}_{eq})_{\text{from A}}$ = 6 ft (1 pt)

b) On the artwork provided, complete the free body diagram for bar ABCDE. Use the F_{eq} determined above in your free body diagram. **(2 pts)**



c) Clearly write the equilibrium equations and solve for the reactions at the pin support A and the roller support B. Express your solution in vector form. **(12 pts)**

$$\underline{\underline{\Sigma M_A = 0}} = -5\,\text{lbs}\left(\tfrac{4}{5}\right)(2\,\text{ft}) - (8\,\text{lbs})(6\,\text{ft}) - (42\,\text{ft-lbs}) + E_y(14\,\text{ft})$$

$$\therefore\ E_y = 7\,\text{lbs.}$$

$$\underline{\underline{\Sigma F_x = 0}} = A_x - (5\,\text{lbs})\left(\tfrac{3}{5}\right)$$

$$\therefore\ A_x = 3\,\text{lbs}$$

$$\Sigma F_y = 0 = A_y + \overset{7\,\text{lbs}}{E_y} - (5\,\text{lbs})\left(\tfrac{4}{5}\right) - (8\,\text{lbs})$$

$$\therefore\ A_y = 5\,\text{lbs.}$$

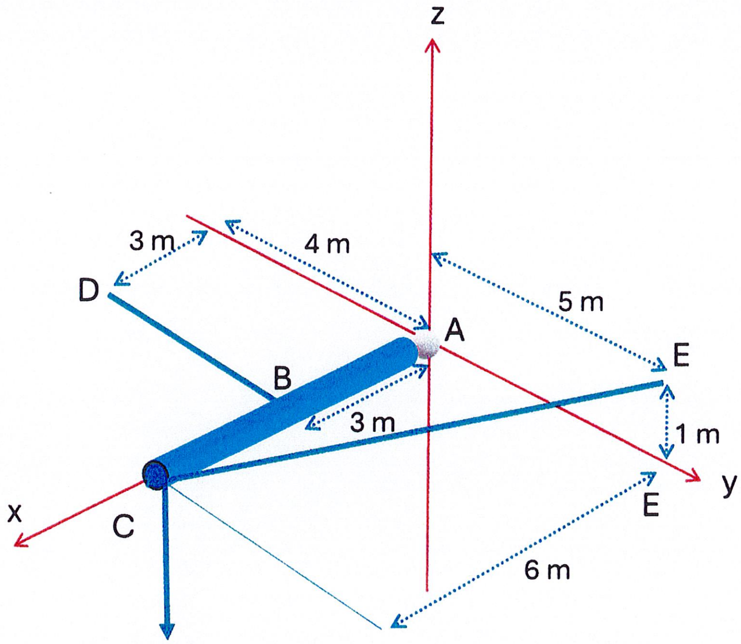
$\bar{F}_B$ = ___7___ $\hat{\jmath}$ ~~kN~~ lbs **(6 pts)** $\bar{F}_A$ = ___3___ $\hat{\imath}$ + ___5___ $\hat{\jmath}$ ~~kN~~ lbs **(6 pts)**

d) If the couple at D were removed from bar ABCDE, what qualitative effect would this have on the <u>magnitudes</u> (i.e., neglect any sign changes) of the reactions (no work need be shown)? **(3 pts)**

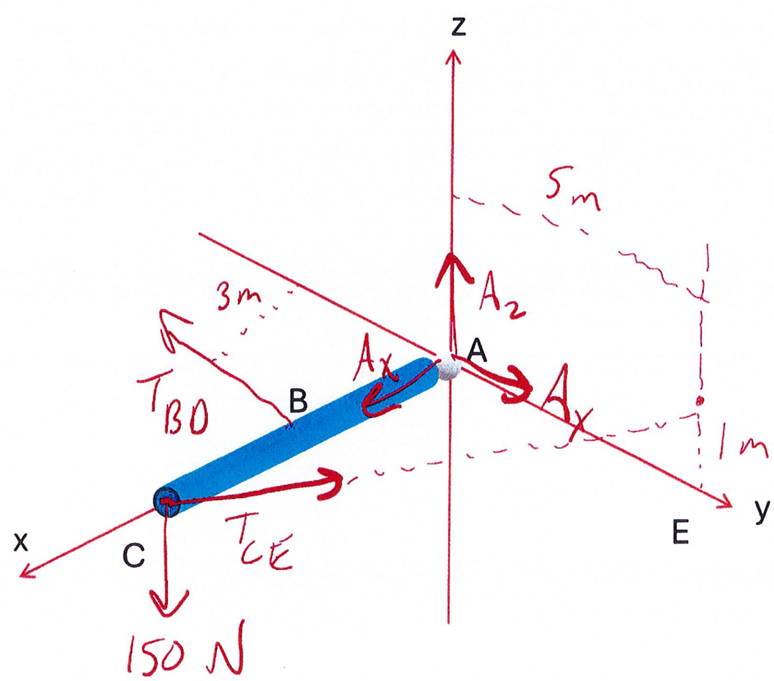
A_x would:	**increase**	**remain the same**	**decrease**	**(circle one)**	**(1 pt)**
A_y would:	**increase**	**remain the same**	**decrease**	**(circle one)**	**(1 pt)**
E_y would:	**increase**	**remain the same**	**decrease**	**(circle one)**	**(1 pt)**

PROBLEM 3. (20 points)

GIVEN: A pole of negligible mass is held in static equilibrium by a ball-and-socket support at point **A** and two cables (**BD** and **CE**) attached to walls at points **D** and **E**. A 150 N load is applied on the pole at x=6 m.



a) Complete the free body diagram of the pole using the artwork below. **(2 pts)**



b) Write expressions for tension vectors T_{BD} and T_{CE} acting on the pole using their unknown magnitudes and known unit vectors. The applied load is shown as an example. Please express your result in reduced fractional form or decimal form. **(4 pts)**

$$\overline{T}_{BD} = T_{BD}\left[\frac{0\vec{i} - 4\vec{j} + 0\vec{k}}{(0^2 + (-4)^2 + 0)^{1/2}}\right] = T_{BD}\left[-1\vec{j}\right]$$

$$\overline{T}_{CE} = T_{CE}\left[\frac{-6\vec{i} + 5\vec{j} + 1\vec{k}}{\left[(-6)^2 + 5^2 + 1^2\right]^{1/2}}\right] = T_{CE}\left[\frac{-6\vec{i}}{\sqrt{62}} + \frac{5\vec{j}}{\sqrt{62}} + \frac{1\vec{k}}{\sqrt{62}}\right]$$

$\overline{T}_{BD} = \lvert\overrightarrow{T_{BD}}\rvert * [($ 0 $)\hat{\imath} + ($ -1 $)\hat{\jmath} + ($ 0 $)\hat{k}]$N		(2 pts)
$\overline{T}_{CE} = \lvert\overrightarrow{T_{CE}}\rvert * [($ $-6/\sqrt{62}$ $)\hat{\imath} + ($ $5/\sqrt{62}$ $)\hat{\jmath} + ($ $1/\sqrt{62}$ $)\hat{k}]$N		(2 pts)
Applied Load $= 150 * [($ 0 $)\hat{\imath} + ($ 0 $)\hat{\jmath} + ($ - 1 $)\hat{k}]$N		(example)

c) Determine the <u>magnitudes</u> of the tension in cables T_{BD} and T_{CE}. **(8 pts)**

$$\Sigma\overline{M}_o = \overline{0} = \left(\vec{r}_{AB} \times \overline{T}_{BD}\right) + \left(\vec{r}_{AC} \times \overline{T}_{CE}\right) + \left(\vec{r}_{AC} \times -150\vec{k}\right)$$

$$\overline{0} = \left[(3\vec{i}) \times (-\vec{j})T_{BD}\right] + \left[(6\vec{i}) \times \left(\frac{-6\vec{i}}{\sqrt{62}} + \frac{5\vec{j}}{\sqrt{62}} + \frac{1\vec{k}}{\sqrt{62}}\right)T_{CE}\right]$$
$$+ \left[(6\vec{i}) \times (-150\vec{k})\right]$$

$$\overline{0} = -3T_{BD}\vec{k} + \left[\frac{30}{\sqrt{62}}\vec{k} - \frac{6}{\sqrt{62}}\vec{j}\right]T_{CE} + 900\vec{j}$$

$$\Sigma M_y = 0 = -\frac{6}{\sqrt{62}}T_{CE} + 900 \qquad T_{CE} = \frac{900\sqrt{62}}{6} = 150\sqrt{62}\text{ N} = \boxed{1181\text{ N}}$$

$$\Sigma M_z = 0 = -3T_{BD} + \frac{30}{\sqrt{62}}T_{CE} \qquad T_{BD} = \frac{10}{\sqrt{62}}T_{CE} = \boxed{1500\text{ N}}$$

$\lvert\vec{T}_{BD}\rvert =$ 1500 N		(4 pts)
$\lvert\vec{T}_{CE}\rvert =$ 1181 N		(4 pts)

d) At point **A**, determine the reactions at **A** and express as a vector. **(6 pts)**

$$\Sigma F_x = 0 = A_x - \frac{6}{\sqrt{62}} T_{CE} \qquad A_x = \frac{6}{\sqrt{62}}(1181) = \boxed{900\ N}$$

$$\Sigma F_y = 0 = A_y - T_{BD} + \frac{5}{\sqrt{62}} T_{CE} \qquad A_y = 1500 - \frac{5}{\sqrt{62}}(1181)$$

$$A_y = \boxed{750\ N}$$

$$\Sigma F_z = 0 = A_z + \frac{1}{\sqrt{62}} T_{CE} - 150 \qquad A_z = 150 - \frac{1181}{\sqrt{62}} = \boxed{0\ N}$$

$$\vec{A} = 900\,\vec{i} + 750\,\vec{j} + 0\,\vec{k}$$

$\vec{A} =$ [(900) $\hat{\imath}$ + (750) $\hat{\jmath}$ + (0) $\hat{k}$] N. **(6 pts)**